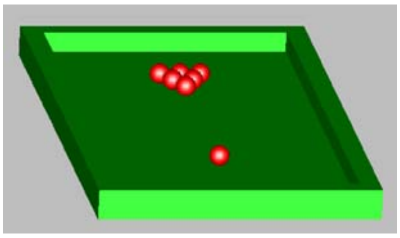
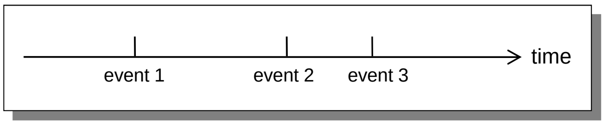
Discrete Events and Hybrid Systems



Picture: Courtesy Hilding Elmqvist

Events

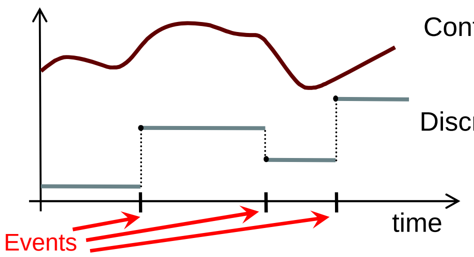
Events are ordered in time and form an event history



- A *point* in time that is instantaneous, i.e., has zero duration
- An *event condition* that switches from false to true in order for the event to take place
- A set of *variables* that are associated with the event, i.e. are referenced or explicitly changed by equations associated with the event
- Some *behavior* associated with the event, expressed as *conditional equations* that become active or are deactivated at the event. *Instantaneous equations* is a special case of conditional equations that are only active *at* events.

Hybrid Modeling

Hybrid modeling = continuous-time + discrete-time modeling



Continuous-time

```
Real x;
Voltage v;
Current i;
```

Discrete-time

```
discrete Real x;
Integer i;
Boolean b;
```

- A *point* in time that is instantaneous, i.e., has zero duration
- An *event condition* so that the event can take place
- A set of *variables* that are associated with the event
- Some *behavior* associated with the event,
  e.g. *conditional equations* that become active or are deactivated at the event

Event creation – if

if-equations, if-statements, and if-expressions

```
if <condition> then
  <equations>
elseif <condition> then
  <equations>
else
  <equations>
end if;
```

```
model Diode "Ideal diode"
  extends TwoPin;
  Real s;
  Boolean off;
equation
  off = s < 0;
  if off then
    v=s
  else
    v=0;
  end if;
  i = if off then 0 else s;
end Diode;
```

False if s<0

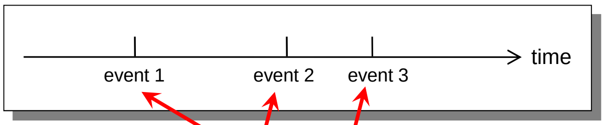
If-equation choosing equation for v

If-expression

Event creation – when

when-equations

when <*conditions*> **then**
<*equations*>
end when;



Equations only active at event times

Time event

```
when time >= 10.0 then
  ...
end when;
```

Only dependent on time, can be scheduled in advance

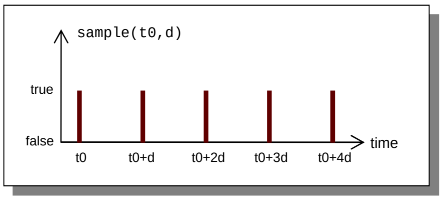
State event

```
when sin(x) > 0.5 then
  ...
end when;
```

Related to a state. Check for zero-crossing

Generating Repeated Events

The call `sample(t0,d)` returns true and triggers events at times `t0+i*d`, where `i=0,1`, …

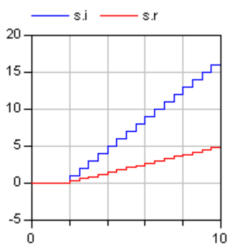


Variables need to be discrete

```
model SamplingClock
  Integer i;
  discrete Real r;
equation
  when sample(2,0.5) then
    i = pre(i)+1;
    r = pre(r)+0.3;
  end when;
end SamplingClock;
```

Creates an event after 2 s, then each 0.5 s

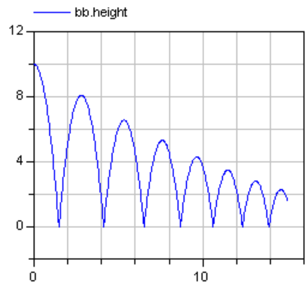
pre(...) takes the previous value before the event.



Reinit – Discontinuous Changes

The value of a *continuous-time* state variable can be instantaneously changed by a `reinit`-equation within a `when`-equation

```
model BouncingBall "the bouncing ball model"
  parameter Real g=9.81;   //gravitational acc.
  parameter Real c=0.90;   //elasticity constant
  Real height(start=10),velocity(start=0);
equation
  der(height) = velocity;
  der(velocity)=-g;
  when height<0 then
    reinit(velocity, -c*velocity);
  end when;
end BouncingBall;
```

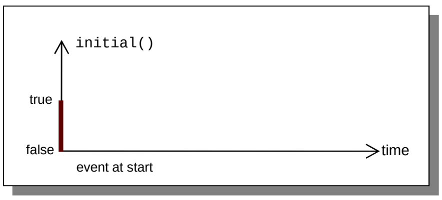


Initial conditions

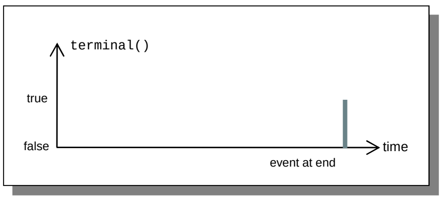
Reinit "assigns" continuous-time variable `velocity` a new value

`initial` and `terminal` events

Initialization actions are triggered by `initial()`



Actions at the end of a simulation are triggered by `terminal()`



Terminating a Simulation

There terminate() function is useful when a wanted result is achieved and it is no longer useful to continue the simulation. The example below illustrates the use:

terminate

```
model terminationModel
  Real y;
equation
  y = time;
  when y >5 then
    terminate("The time has elapsed 5s");
  end when;
end terminationMode;
```

Simulation ends before reaching time 10

```
simulate(terminationModel, startTime = 0, stopTime = 10)
```


Expressing Event Behavior in Modelica

if-equations, if-statements, and if-expressions express different behavior in different operating regions

if *<condition>* **then**
  <equations>
elseif *<condition>* **then**
  <equations>
else
  <equations>
end if;

```
model Diode "Ideal diode"
  extends TwoPin;
  Real s;
  Boolean off;
equation
  off = s < 0;
  if off then
    v=s
  else
    v=0;
  end if;
  i = if off then 0 else s;
end Diode;
```

when-equations become active at events

when *<conditions>* **then**
  <equations>
end when;

```
equation
  when x > y.start then
    ...
```


Event Priority

Erroneous multiple definitions, single assignment rule violated

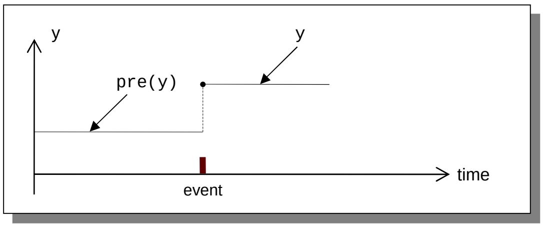
```
model WhenConflictX    // Erroneous model: two equations define x
  discrete Real x;
  equation
  when time>=2 then    // When A: Increase x by 1.5 at  time=2
    x = pre(x)+1.5;
  end when;
  when time>=1 then    // When B: Increase x by 1 at   time=1
    x = pre(x)+1;
  end when;
end WhenConflictX;
```

Using event priority to avoid erroneous multiple definitions

```
model WhenPriorityX
  discrete Real x;
equation
  when time>=2 then          // Higher priority
    x = pre(x)+1.5;
  elsewhen time>=1 then      // Lower priority
    x = pre(x)+1;
  end when;
end WhenPriorityX;
```


Obtaining Predecessor Values of a Variable Using `pre()`

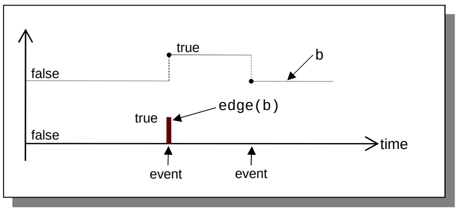
At an event, `pre(y)` gives the previous value of y immediately before the event, except for event iteration of multiple events at the same point in time when the value is from the previous iteration



- The variable *y* has one of the basic types `Boolean`, `Integer`, `Real`, `String`, or `enumeration`, a subtype of those, or an array type of one of those basic types or subtypes
- The variable y is a discrete-time variable
- The `pre` operator can *not* be used within a function

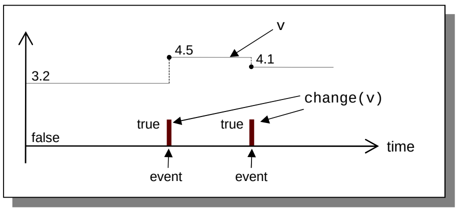
Detecting Changes of Boolean Variables Using `edge()` and `change()`

Detecting changes of boolean variables using `edge()`



The expression `edge(b)` is true at events when b switches from false to true

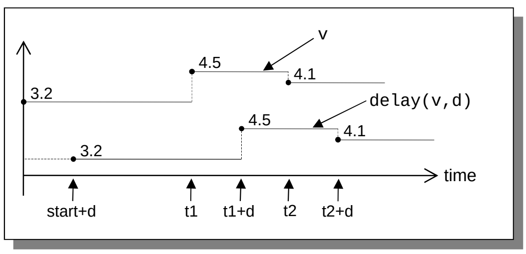
Detecting changes of discrete-time variables using `change()`



The expression `change(v)` is true at instants when v changes value

Creating Time-Delayed Expressions

Creating time-delayed expressions using `delay()`

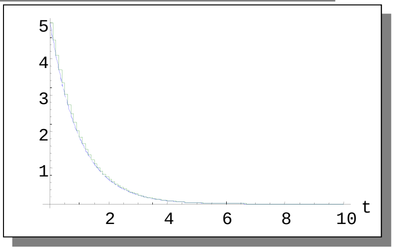


In the expression `delay(v,d)` *v* is delayed by a delay time *d*

A Sampler Model

```
model Sampler
  parameter Real sample_interval = 0.1;
  Real x(start=5);
  Real y;
equation
  der(x) = -x;
  when sample(0, sample_interval) then
    y = x;
  end when;
end Sampler;
```

```
simulate(Sampler, startTime = 0, stopTime = 10)
plot({x,y})
```

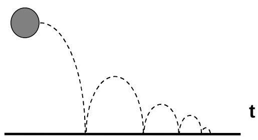


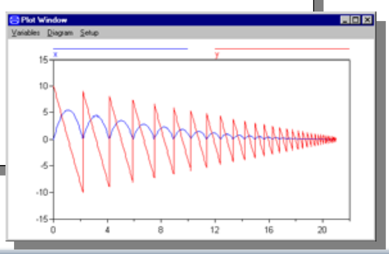
Discontinuous Changes to Variables at Events via When-Equations/Statements

The value of a *discrete-time* variable can be changed by placing the variable on the left-hand side in an equation within a `when`-equation, or on the left-hand side of an assignment statement in a `when`-statement

The value of a *continuous-time* state variable can be instantaneously changed by a `reinit`-equation within a `when`-equation

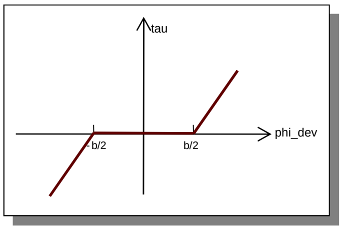
```
model BouncingBall "the bouncing ball model"
  parameter Real g=9.18;   //gravitational acc.
  parameter Real c=0.90;   //elasticity constant
  Real x(start=0),y(start=10);
equation
  der(x) = y;
  der(y)=-g;
  when x<0 then
    reinit(y, -c*y);
  end when;
end BouncingBall;
```



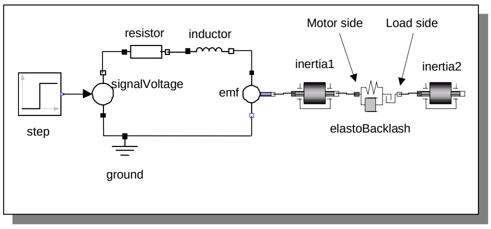


A Mode Switching Model Example

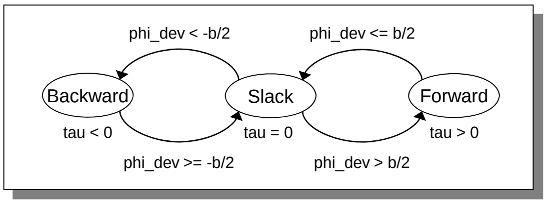
Elastic transmission with slack



DC motor transmission with elastic backlash



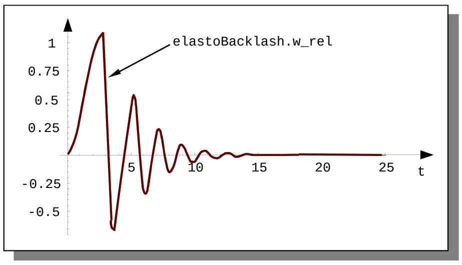
A finite state automaton `SimpleElastoBacklash` model



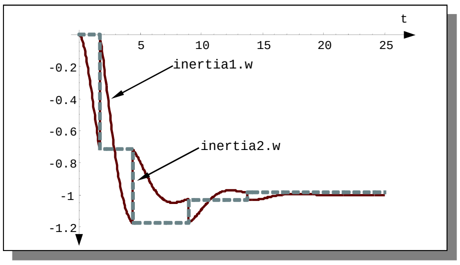
A Mode Switching Model Example cont'

```
partial model SimpleElastoBacklash
  Boolean backward, slack, forward;  // Mode variables
  parameter Real  b                  "Size of backlash region";
  parameter Real  c = 1.e5           "Spring constant (c>0), N.m/rad";
  Flange_a        flange_a           "(left) driving flange - connector";
  Flange_b        flange_b           "(right) driven flange - connector";
  parameter Real  phi_rel0 = 0       "Angle when spring exerts no torque";
  Real         phi_rel               "Relative rotation angle betw. flanges";
  Real         phi_dev               "Angle deviation from zero-torque pos";
  Real         tau                   "Torque between flanges";
equation
  phi_rel   = flange_b.phi - flange_a.phi;
  phi_dev   = phi_rel - phi_rel0;
  backward  = phi_rel < -b/2;                // Backward angle gives torque tau<0
  forward   = phi_rel > b/2;                 // Forward angle gives torque tau>0
  slack     = not (backward or forward);     // Slack angle gives no torque
  tau = if forward then                      // Forward angle gives
           c*(phi_dev – b/2)                 // positive driving torque
        else (if backward then               // Backward angle gives
                c*(phi_dev + b/2)            // negative braking torque
              else                           // Slack gives
                0);                          // zero torque
end SimpleElastoBacklash
```


A Mode Switching Model Example cont’



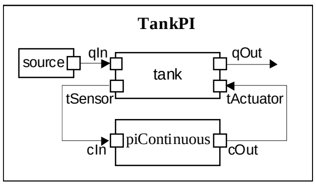
Relative rotational speed between the flanges of the `Elastobacklash` transmission



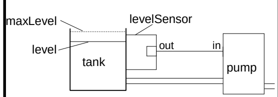
We define a model with less mass in `inertia2(J=1)`, no damping `d=0`, and weaker string constant `c=1e-5`, to show even more dramatic backlash phenomena

The figure depicts the rotational speeds for the two flanges of the transmission with elastic backlash

Water Tank System with PI Controller




```
model TankPI
  LiquidSource             source(flowLevel=0.02);
  Tank                     tank(area=1);
  PIcontinuousController piContinuous(ref=0.25);
equation
  connect(source.qOut, tank.qIn);
  connect(tank.tActuator, piContinuous.cOut);
  connect(tank.tSensor, piContinuous.cIn);
end TankPI;
```




```
model Tank
  ReadSignal tOut;  // Connector, reading tank level
  ActSignal  tInp;  // Connector, actuator controlling input flow
  parameter Real flowVout = 0.01;       // [m3/s]
  parameter Real area = 0.5;            // [m2]
  parameter Real flowGain = 10;         // [m2/s]
  Real h(start=0);                      // tank level [m]
  Real qIn;                   // flow through input valve[m3/s]
  Real qOut;                  // flow through output valve[m3/s]
equation
  der(h)=(qIn-qOut)/area;               // mass balance equation
  qOut=if time>100 then flowVout else 0;
  qIn = flowGain*tInp.act;
  tOut.val = h;
end Tank;
```


Water Tank System with PI Controller – cont’

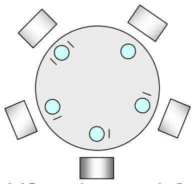
```
partial model BaseController
  parameter Real Ts(unit = "s") = 0.1   "Time period between discrete samples";
  parameter Real K = 2                  "Gain";
  parameter Real T(unit = "s") = 10     "Time constant";
  ReadSignal cIn                        "Input sensor level, connector";
  ActSignal  cOut                       "Control to actuator, connector";
  parameter Real ref                    "Reference level";
  Real error                            "Deviation from reference level";
  Real outCtr                           "Output control signal";
equation
  error = ref - cIn.val;
  cOut.act = outCtr;
end BaseController;
```

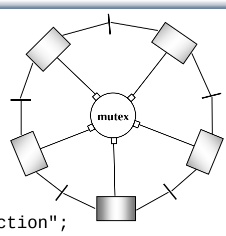
```
model PIDcontinuousController
  extends BaseController(K = 2, T = 10);
  Real  x;
  Real  y;
equation
  der(x) = error/T;
  y      = T*der(error);
  outCtr = K*(error + x + y);
end PIDcontinuousController;
```

```
model PIdiscreteController
  extends BaseController(K = 2, T = 10);
  discrete Real x;
equation
  when sample(0, Ts) then
    x = pre(x) + error * Ts / T;
    outCtr = K * (x+error);
  end when;
end PIdiscreteController;
```


Concurrency and Resource Sharing

Dining Philosophers Example






```
model DiningTable
  parameter Integer n = 5  "Number of philosophers and forks";
  parameter Real sigma = 5 " Standard deviation for the random function";
  // Give each philosopher a different random start seed
  // Comment out the initializer to make them all hungry simultaneously.
  Philosopher phil[n](startSeed=[1:n,1:n,1:n], sigma=fill(sigma,n));
  Mutex       mutex(n=n);
  Fork        fork[n];
equation
  for i in 1:n loop
    connect(phil[i].mutexPort, mutex.port[i]);
    connect(phil[i].right, fork[i].left);
    connect(fork[i].right, phil[mod(i, n) + 1].left);
  end for;
end DiningTable;
```