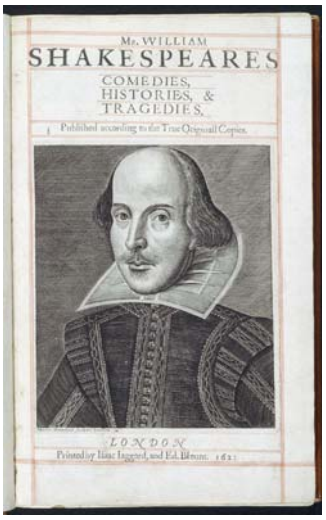
An Introduction to Shakespeare

The Dynamics of Shakespearian Characters



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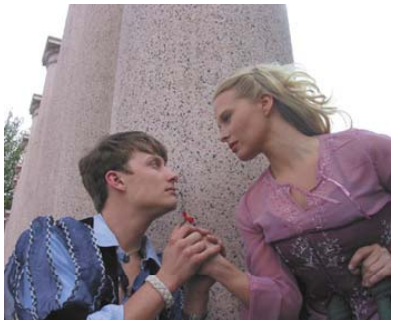
Problem formulation

Romeo is in love with Julia



How to model such complicated systems?

A Mathematical Description of Romeo

$$\frac{dx}{dt} = ax + by$$

where x is the Romeo love for Julia

x > 0 

x = 0 Romeo doesn't care about Julia

x < 0 Romeo hates Julia

a and b describes Romeo romantic style

Romeo's Romantic Styles

$$\frac{dx}{dt} = ax + by$$

- *a*=0 (out of touch with own feelings)
- *b*=0 (oblivious to other's feelings)
- *a*>0, *b*>0 (eager beaver)
- *a*>0, *b*<0 (narcissistic nerd)
- *a*<0, *b*>0 (cautious lover)
- *a*<0, *b*<0 (hermit)

What about Julia?

She has her own style

$$\frac{dy}{dt} = cy + dx$$

y > 0

y = 0 Julia doesn't care about Romeo

y < 0 Julia hates Romeo

c and d describes Julia romantic style

Putting Romeo and Julia Together

Putting Romeo and Julia Together – cont'

$$\begin{cases} \frac{dx}{dt} = ax + by \\ \frac{dy}{dt} = cx + dy \end{cases}$$

The mathematician's remark:

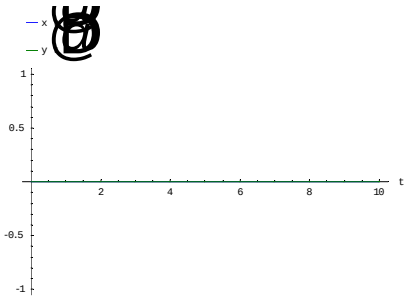
William Shakespeare needed 25 674 words and 1938 paragraphs to explain this

4 parameters with 3 choices for each gives 81 different romantic pairings

Love needs initial conditions

$$\begin{cases} \frac{dx}{dt} = ax + by \\ \frac{dy}{dt} = cx + dy \end{cases}$$

$$x[0] = 0; \quad y[0] = 0$$

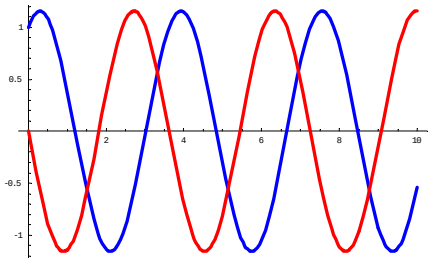


Love Affair #1

Do opposites attract?

- Take $c = -b$ and $d = -a$

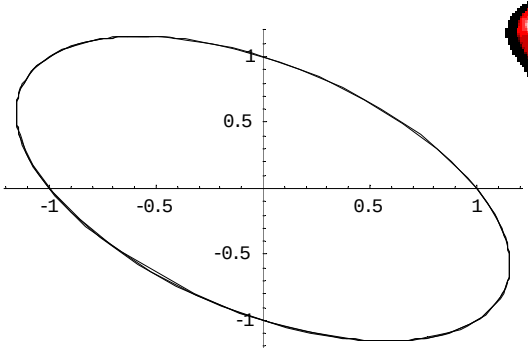
$$\begin{cases} \dfrac{dx}{dt} = ax + by \\ \dfrac{dy}{dt} = -bx - ay \end{cases}$$

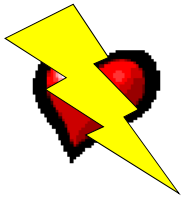


• Romeo's feelings show that he really likes Julia but as soon as his affections are not met, he gets discouraged and cools down

• In the true spirit of a woman, Julia starts to like him as soon as his attention fades

Love Affair #1 – Parametric Plot



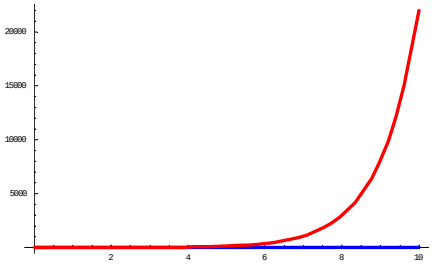


Love Affair #1 – cont'

Love Affair #2 - Romeo the Robot

- Take $a = 0$, $b = 0$ x[0]=0

$$\begin{cases} \dfrac{dx}{dt} = 0 \\ \dfrac{dy}{dt} = cx + dy \end{cases}$$

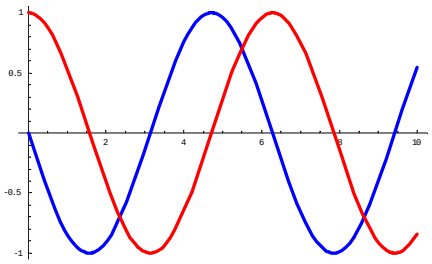


- Can be either love or hate depending on the sign of y, c and d.
- Romeo's indifference drives Julia insane with passion
- Her feelings never dies

Love Affair #3

- $a = 0$, $b = -1$
- c = 1, d = 0

$$\begin{cases} \dfrac{dx}{dt} = -by \\ \dfrac{dy}{dt} = x \end{cases}$$



• The same never ending cycle from Love affair #1

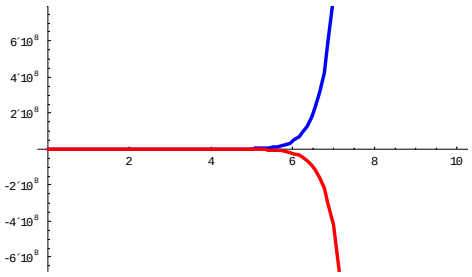
• At t=0 Julia likes Romeo and Romeo dislikes Julia

• Again their feelings oscillate, neither sure they love each other

Love Affair #4

- $a = 2$, $b = -2$
- c = -1, d = 1

$$\begin{cases} \dfrac{dx}{dt} = 2x - 2y \\ \dfrac{dy}{dt} = -x + y \end{cases}$$



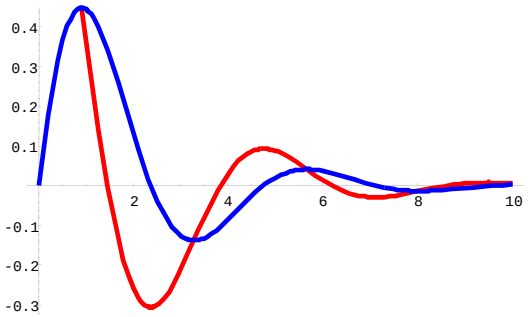
• During the first 7 years of knowledge R&J feel indifference to each other

• At the same time R realizes his love for J, J realizes that he is repulsed of him

Love Affair #5

- $a = 0$, $b = -2$
- c = 1, d = -1

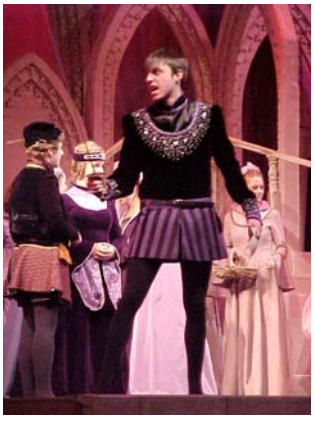
$$\begin{cases} \dfrac{dx}{dt} = -2y \\ \dfrac{dy}{dt} = x - y \end{cases}$$



• Julia's love for Romeo is out of control. We tried to help her by damping her reactions with negative values of y. Unfortunately the damping was contagious and sadly their feelings for each other died.

Love Affair #6

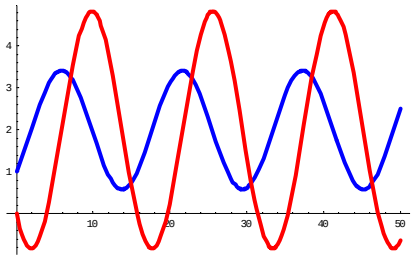
Tybalt the damper



Love Affair #5 -cont'

Love Affair #6

$$\begin{cases} \dfrac{dx}{dt} = -0.2(y-2) \\ \dfrac{dy}{dt} = 0.8(x-2) \end{cases}$$

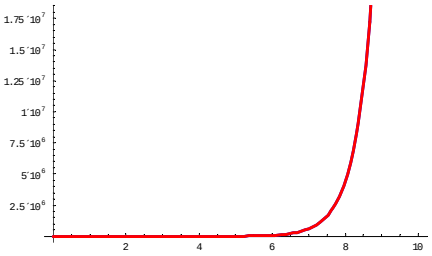


• Julia responds to Romeo's love but when she becomes too affectionate his love decreases

Which Version was Chosen by Shakespeare?

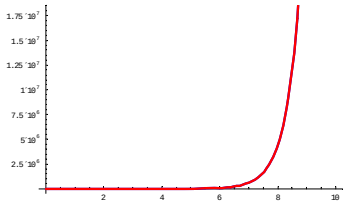
- $a = 1$, $b = 1$
- c = 1, d = 1

$$\begin{cases} \dfrac{dx}{dt} = ax + by \\ \dfrac{dy}{dt} = cx + dy \end{cases}$$

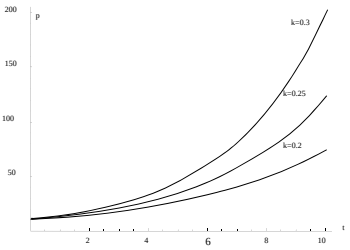


Shakespeare's Version

Exchanging the Model



In this situation we need to find another model



The Rabbit Population Growth Model

$$\frac{dp}{dt} = kp$$

p = number of individuals in a population at a time *t*

k = reproduction coefficient

What if ?

- Romeo has a mistress, Rowena
- Rowena and Julia don't know about one another
- Romeo responds to each with the same romantic style (same *a* and *b*)
- Rowena's hate has the same effect on his feelings for Julia as does Julia's love, and vice versa

Love Triangle Equations ?

$$\frac{dx_{Juliet}}{dt} = ax_{Juliet} + b(y - z)$$

$$\frac{dy}{dt} = cx_{Juliet} + dy$$

$$\frac{dx_{Rowena}}{dt} = ax_{Rowena} + b(z - y)$$

$$\frac{dz}{dt} = ex_{Rowena} + fz$$

- System is 4D (4 variables)
- There are 6 parameters
- System is linear (no chaos)

Romeo's fate ?

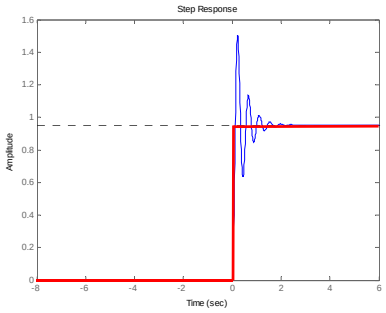
- Averaged over all romantic styles (64 combinations of parameters) and 64 initial conditions:
  - 37% loves Julia & hates Rowena
  - 37% loves Rowena & hates Julia
  - 6% loves both (2% everyone in love)
  - 6% hates both (2% everyone in hate)
  - 14% apathy (10% everyone apathetic)
- Anything can happen!

How about me ?

Since I'm happily married the previous model entered into a stable state.

```
num=1;
den=[1 5 10];
Kp=200;

[numCL,denCL]=cloop(Kp*num,den, -1);
t=0:0.01:6;
step(numCL, denCL,t)
```



Let's analyze my happiness instead

Happiness model ?

$$\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + \omega^2 x = F(t)$$

Damping Oscillation External Force

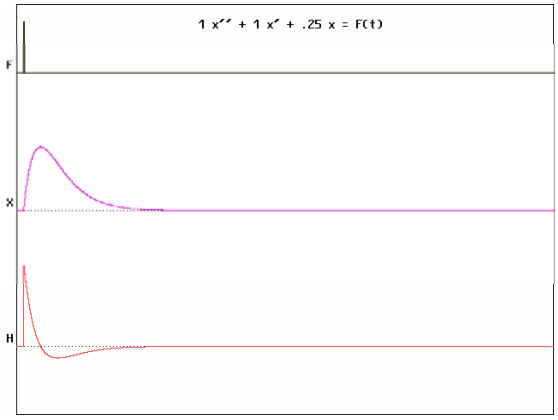
Happiness $H = \frac{dx}{dt}$

What is x ?

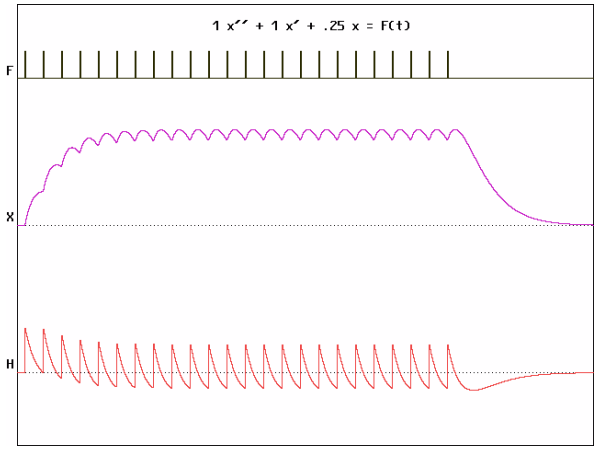
$$\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + \omega^2 x = F(t)$$

- x = integral of H
- x is what others perceive
- In the love model x is what the other feels
- H (your happiness) must average to zero (with positive damping)
- x does not average to zero

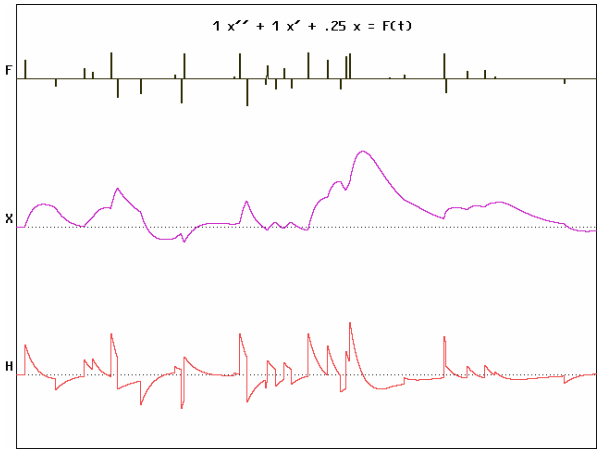
Winning the lottery ?



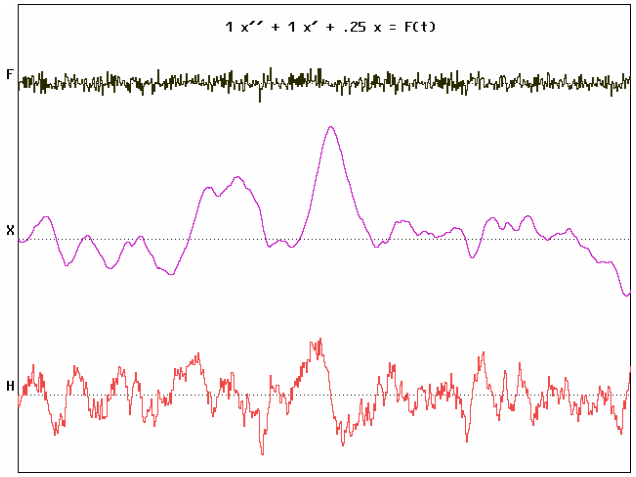
Drug or Other Addiction ?



Random Events ?



Real Life ?



Some implications

- Constant happiness is an unrealistic goal.
- Others see less volatility in you and often wrongly conclude how you feel.
- Rome and Julia might not be so happy as it looks.
- Individuals can be categorized by their values of β and ω.
- Long prison terms may be ineffective.

Conclusions

William Shakespeare's drama is complex

but

simple models might suffice

References and Acknowledgements

Thanks to J.C Sprott *Department of Physics University of Wisconsin - Madison*

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