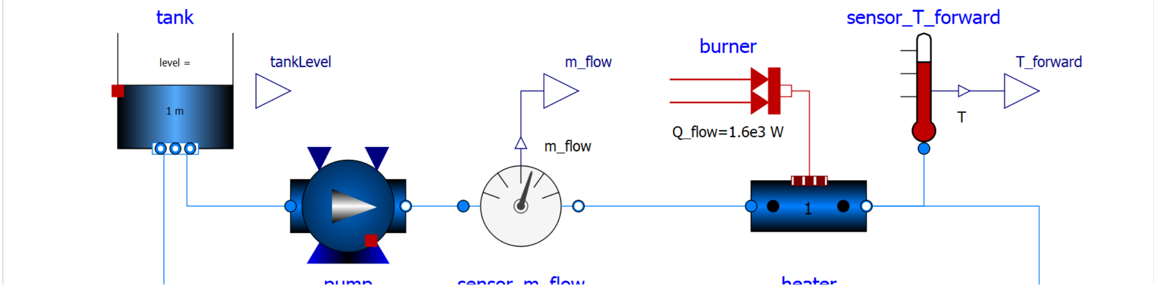

Efficient Minimal Tearing of Hybrid Algebraic Loops for Large-scale System Simulation

OpenModelica Annual Workshop 2020

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Table of Contents

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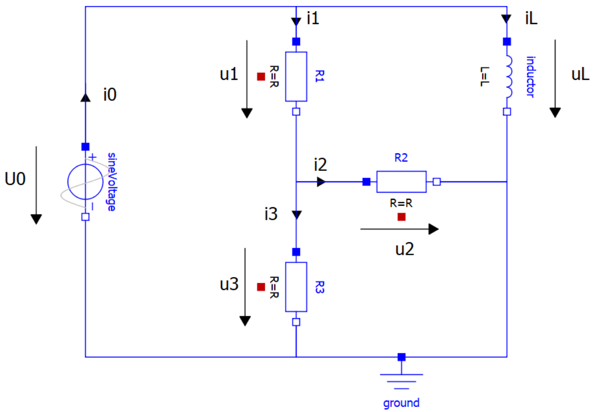
Efficient Minimal Tearing of Hybrid Algebraic Loops for Large-scale System Simulation

Modelica Compiler Overview

Modelica Compiler Overview

How to get a mathematical representation from a model



Equations

$$u_0 = A\sin(2\pi f t) \qquad u_0 = u_1 + u_3$$

$$u_1 = R_1 \cdot i_1 \qquad u_L = u_1 + u_2$$

$$u_2 = R_2 \cdot i_2 \qquad u_3 = u_2$$

$$u_3 = R_3 \cdot i_3 \qquad i_0 = i_1 + i_L$$

$$u_L = L \cdot \dot{i}_L \qquad i_1 = i_2 + i_3$$

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Modelica Compiler Overview

Symbolic Transformation

Steps to perform

- Causalization
  - Assign each variable to exactly one equation
- Matching and sorting
  - Find strong components and sorting
- Adjacency matrix and structural regularity
  - BLT transformation

Group variables

$$\underline{0} = \underline{f}(\underline{x}(t), \underline{\dot{x}}(t), \underline{y}(t), \underline{u}(t), \underline{p}, t)$$

$$\underline{\dot{x}}(t) = \begin{pmatrix} \dot{i}_L \end{pmatrix} \quad \underline{x}(t) = \begin{pmatrix} i_L \end{pmatrix}$$

$$\underline{y}(t) = \begin{pmatrix} u_0 \\ u_1 \\ u_2 \\ u_3 \\ u_L \\ i_0 \\ i_1 \\ i_2 \\ i_3 \end{pmatrix} \quad \underline{p} = \begin{pmatrix} A \\ f \\ R_1 \\ R_2 \\ R_3 \\ L \end{pmatrix} \quad \underline{u}(t) = \begin{pmatrix} \end{pmatrix}$$

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Modelica Compiler Overview
Symbolic Transformation

- Matching: Assign variables to equations
- Sorting: Construct directed graph
- Get ordered state form

Symbolic transformation

$$\underline{0} = f(\underline{x}(t), \underline{z}(t), \underline{u}(t), t), \quad \underline{z}(t) = \begin{pmatrix} \dot{\underline{x}}(t) \\ \underline{y}(t) \end{pmatrix}$$

$$\underline{z}(t) = \underline{g}(\underline{x}(t), \underline{u}(t), \underline{p}, t)$$

$$\dot{\underline{x}}(t) = \underline{h}(\underline{x}(t), \underline{u}(t), \underline{p}, t)$$
$$\underline{y}(t) = \underline{k}(\underline{x}(t), \underline{u}(t), \underline{p}, t)$$

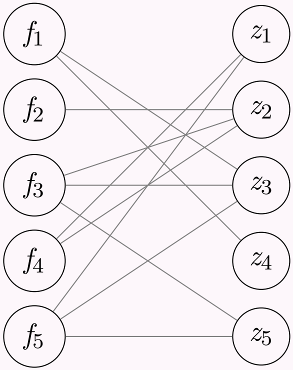
Modelica Compiler Overview

Matching & Sorting

Adjacency matrix

$$
\begin{array}{c} \\ f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{array}
\begin{array}{c} \begin{array}{ccccc} z_1 & z_2 & z_3 & z_4 & z_5 \end{array} \\
\begin{pmatrix} 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 \end{pmatrix} \end{array}
$$

Bipartite Graph



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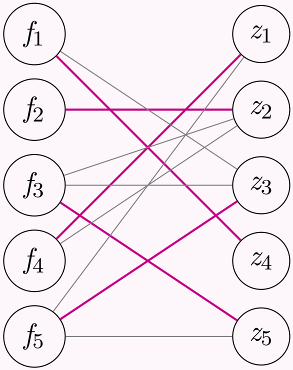
Modelica Compiler Overview

Matching & Sorting

Adjacency matrix

	z_1	z_2	z_3	z_4	z_5
f_1	0	0	1	**1**	0
f_2	0	**1**	0	0	0
f_3	0	1	1	0	**1**
f_4	**1**	1	0	0	0
f_5	1	0	**1**	0	1

Bipartite Graph



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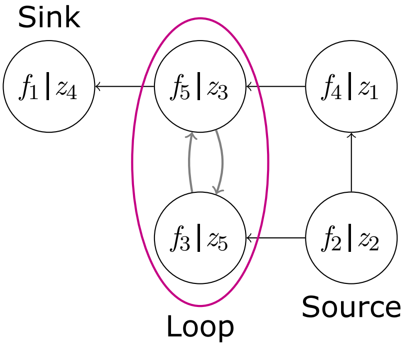
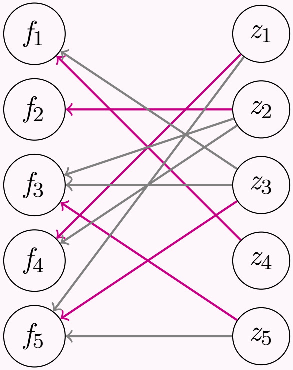
Modelica Compiler Overview

Matching & Sorting



Bipartite Graph



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Modelica Compiler Overview

Block-Lower-Triangular

For our start example we get:

$$
\begin{array}{c} \\ f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_6 \\ f_8 \\ f_{10} \\ f_7 \\ f_5 \\ f_9 \end{array}
\begin{array}{c}
\begin{array}{cccccccccc} u_0 & u_1 & i_1 & u_2 & i_2 & u_3 & i_3 & u_L & \dot{i}_L & i_o \end{array} \\
\begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}
\end{array}
$$

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Efficient Minimal Tearing of Hybrid Algebraic Loops for Large-scale System Simulation

Tearing

Overview

Tearing

Target:

- Efficient computation with numerical solver
- Reduces size of algebraic loops

General Idea

- Assume variables are already known (=Tearing variables)
- Causalize equations with this assumption (=inner equations)
- Remaining Equations are residual equations

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Tearing

Choose good tearing variables

Example

$$u_1 - R_1 \cdot i_1 = 0$$
$$u_2 - R_2 \cdot i_2 = 0$$
$$u_3 - R_3 \cdot i_3 = 0$$
$$u_1 + u_3 = u_0$$
$$u_2 - u_3 = 0$$
$$i_1 - i_2 - i_3 = 0$$

- Find minimal number of iteration variables
  - Mind solvability
  - Problem is NP-hard
- Use heuristics to choose in polynomial time
  - `--tearingMethod = noTearing`
  - `--tearingMethod = minimalTearing`
  - `--tearingMethod = omcTearing`
  - `--tearingMethod = cellier`

Tearing

Choose good tearing variables

Assuem i_3 is known

$$u_3 = R_3 \cdot i_3$$

$$u_1 = u_0 - u_3$$

$$i_1 = \frac{u_1}{R_1}$$

$$u2 = u_3$$

$$i_2 = \frac{u_2}{R_2}$$

Residual equation:

$$0 = i_1 - i_2 - i_3$$

- Try to minimize number of iteration variables
  - Mind solvability
  - Problem is NP-hard
- Use heuristics to choose in polynomial time
  - `--tearingMethod = noTearing`
  - `--tearingMethod = minimalTearing`
  - `--tearingMethod = omcTearing`
  - `--tearingMethod = cellier`

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Tearing

Hybrid Algebraic Loops

What if I want to disable tearing?

- `omcTearing` or `cellier` to time consuming
- Use sparse non-linear solvers
  - That’s what `--daeMode` is doing
- Debugging and library development

Problem

Hybrid algebraic loops need tearing!

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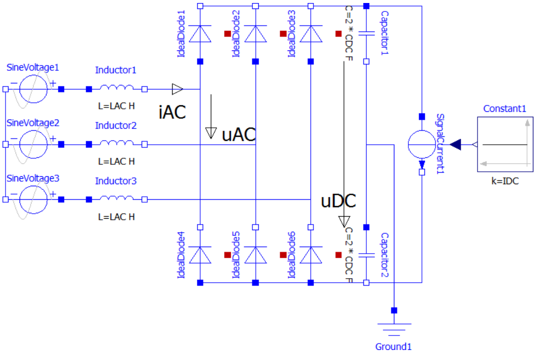
Tearing
Hybrid Algebraic Loops

`Modelica.Electrical.Analog.Examples.Rectifier`



- `IdealDiode1.off, ..., IdealDiode6.off` inside algebraic loops
- "Switching state" type: Boolean

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Tearing

Minimal Tearing

Minimal Tearing

Perform the bare minimum of optimization

Want to causalization of:

- Discrete variables (Boolean, Integer, ...)
- Variables solved inside an algorithm with discrete variables as outputs

General Idea

1. Search all discrete variables, `tearingSelect=never`-variables and variables from algorithms
2. Match found variables to equations of algebraic loop
   $\Rightarrow$ Set as inner variables and equations
3. Remaining variables are iteration variables

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Minimal Tearing
Implementation

`--tearingMethod=minimalTearing`

- implemented in OpenModelica v1.14
  - Cases for when-equations, if-equations and algorithms will be implemented in v1.16
- Performance: Matching on discrete variables of strong component
  - $\mathcal{O}(|V| \cdot |E|)$ with V discrete variables of loop, E connected equations
- Supersede `--tearingMethod=noTearing`

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Thank you for your attention

Questions

Refrences

- Cellier, E. Francio und Kofman, Ernesto. 2006. Continous System Simulation. New York: Springer Science+Business Media Inc., 2006.
- R. Bulatow, Entwicklung einer Fast-Tearing-Methode zur effizienten Simulation großer differential-algebraischer Gleichungssysteme, Bachelor thesis, 2019