Sensitivity Analysis and Non-linear Optimization with OMSens in OpenModelica

Rodrigo Castro – University of Buenos Aires

Adeel Asghar – Research Institutes of Sweden RISE

Peter Fritzson – Linköping University

LINKÖPING UNIVERSITY

UBA Universidad de Buenos Aires

OpenModelica

RISE

Outline

- Introduction
- Sensitivity approaches
- Background & Motivation
- CURVI family
- OMSens
- Conclusion & Future work
- Demonstration

Introduction to Sensitivity Analysis

- Sensitivity of nonlinear systems in the form of ODEs undergo **noticeable dynamic changes** in response to **small perturbations** in the parameters.
- OO- modeling languages (Modelica)
  - Systematic treatment of the problem
  - Clear, unambiguous access to parameters, variables and simulation configuration.
  - Reusable frameworks to manipulate models as black boxes.

Approaches to Sensitivity Analysis

- Individual
  - One parameter perturbed at a time
  - Testing the extreme values of the uncertainty interval pi ±Δpi
  - Non-linear model cannot be assessed
- Simultaneous
  - All possible combinations not feasible (Combinatorial explosion)
  - Find optimal combinations of perturbations **smallest simultaneous perturbations** that produce **largest deviations**

Background and Motivation

- Sensitivity analysis
  - Automatic differentiation [Elsheikh, 2012] [IDASens, 2017]
  - Parameter sweep and solver-based approaches using DAPSK [Wolf et al., 2008] [Petzold et al., 2006]
- Optimization methods
  - Nonlinear optimization routine [Ipopt, 2017]
  - Genetic algorithms [Dymola, 2017] [Thieriot et al., 2011]
  - Collocation methods [Åkesson, 2008] [Bachmann et al., 2012] [Houska et al., 2011]

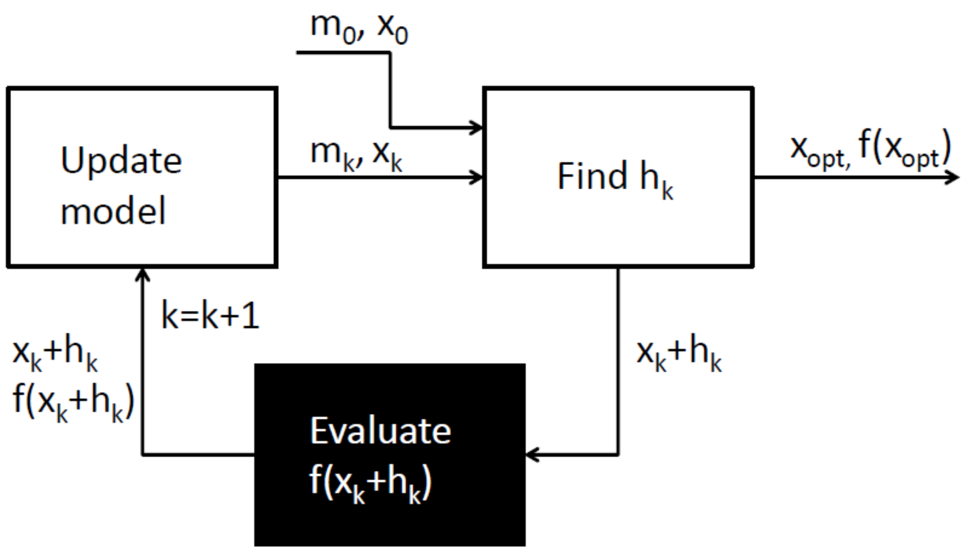
No previous sensitivity analysis Modelica tool provides simultaneous parameter sensitivity analysis as well as being robust since based on the non-linear derivative-free optimization method CURVIF

CURVI Family

- The CURVI family
  - Curvilinear search approach
  - Three versions: CURVIF, CURVIG, CURVIH
  - **F**unction values, function values plus **G**radients, and the latter plus **H**essians.
  - All globally convergent
  - Uses fewer evaluations than other algorithms
- CURVIF: the flavor adopted for **OMSens**
  - Trade-off: favor **robustness**, sacrifice some efficiency
  - Derivative-free methods can either be robust - at the cost of using many function evaluations, e.g. direct searches - or may present convergence problems

CURVIF: Robust derivative free model-building method

- Uses **function values** to on-the-fly build an **interpolation model** within a **trust region** which is dynamically updated
- Can handle goal functions with some **discontinuities** (can occur with Modelica models with **events**)
- Minimizes the number of needed function evaluations by using the model to guide the search

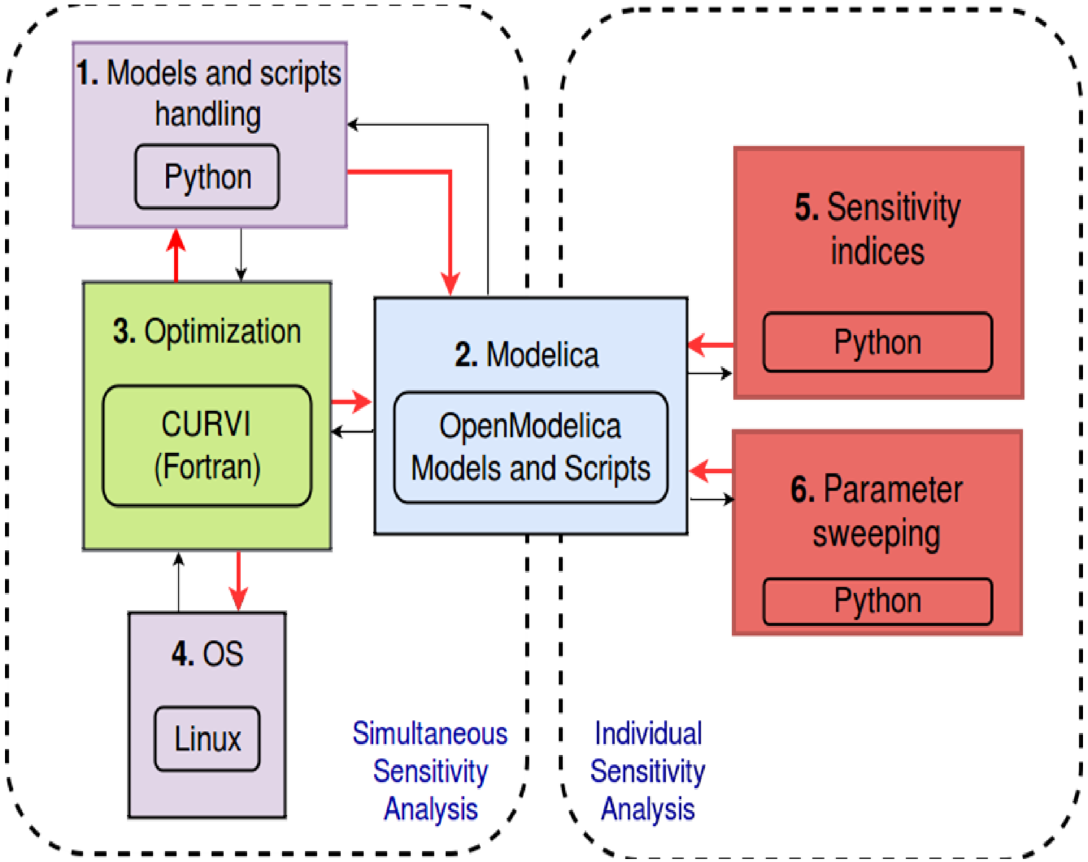


OMSens

- Flexible experimentation platform for different sensitivity analysis strategies, for Modelica models.
- Modularity
  - Split the system into decoupled backend/frontend modules.
- Flexibility
  - Subdivide the backend into modules
  - Expose clear invocation interfaces
- Scalability
  - New modules and features added without compromising previous features

OMSens Parameter Sensitivity Scenarios

- Individual sensitivity analysis
  - Modules 5 and 6 lead their own workflows
    - Invoke individual simulations
    - A new simulation is independent of previous simulation results
- Simultaneous sensitivity analysis
  - **Dynamic exploration** of the parameter space
  - Module 3 (Optimization) leads the workflow
    - Invokes modules 1, 2 and 4
    - Successive simulations requested to Module 2 (Modelica) depend on the simulation results of previous calls
    - Closed loop strategy
  - Find **optimal** perturbation vectors
    - Max impact with Min perturbation



OMSens Individual Sensitivity Analysis

- Computes a **measure of the change** in a chosen variable with respect to **changes in one or more influencing parameters**
- **IDAsens**: provided by OpenModelica [IDASens,2017], uses IDA solver
  - Computes the derivatives of the state variables with respect to each top-level parameter
- **Relative (Rel)** and **Root Mean Square (RMS)**: new, provided by OMSens
  - Compare results between **unperturbed** and **perturbed** runs.
    Rel can be used to **rank** the parameters **affecting** a variable the most (at a specific point in time)
  - **RMS** is similar to **Rel** but focusing on a **time range**

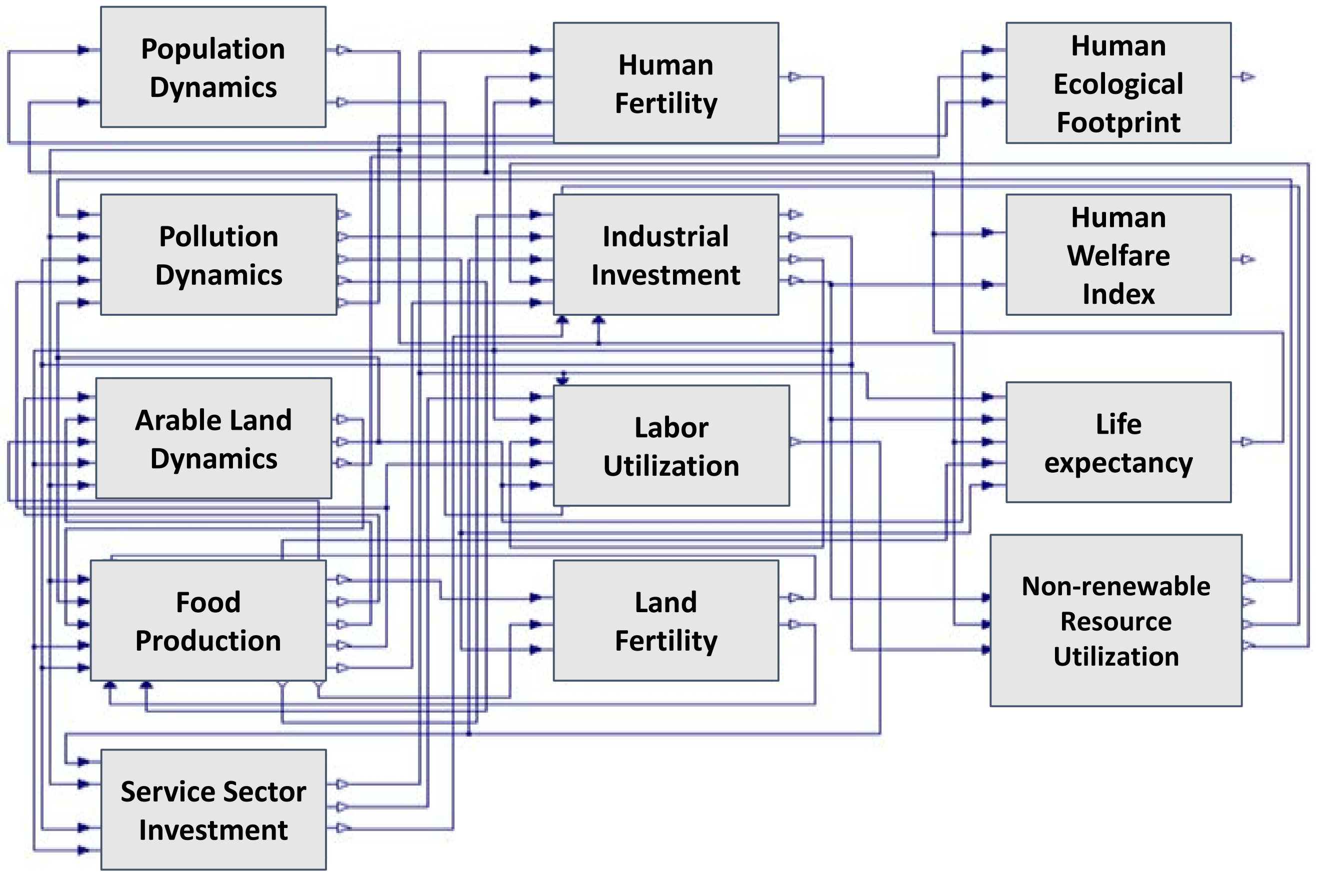
IDASens vs OMSens

- **IDASens** is a combined model/simulation-based approach
  - The model/equation $s = \partial x / \partial p$ is internally calculated and updated throughout the simulation
- **Rel** and **RMS** are purely empirical, simulation-based approaches
  - No model/equation of partial derivatives is ever required

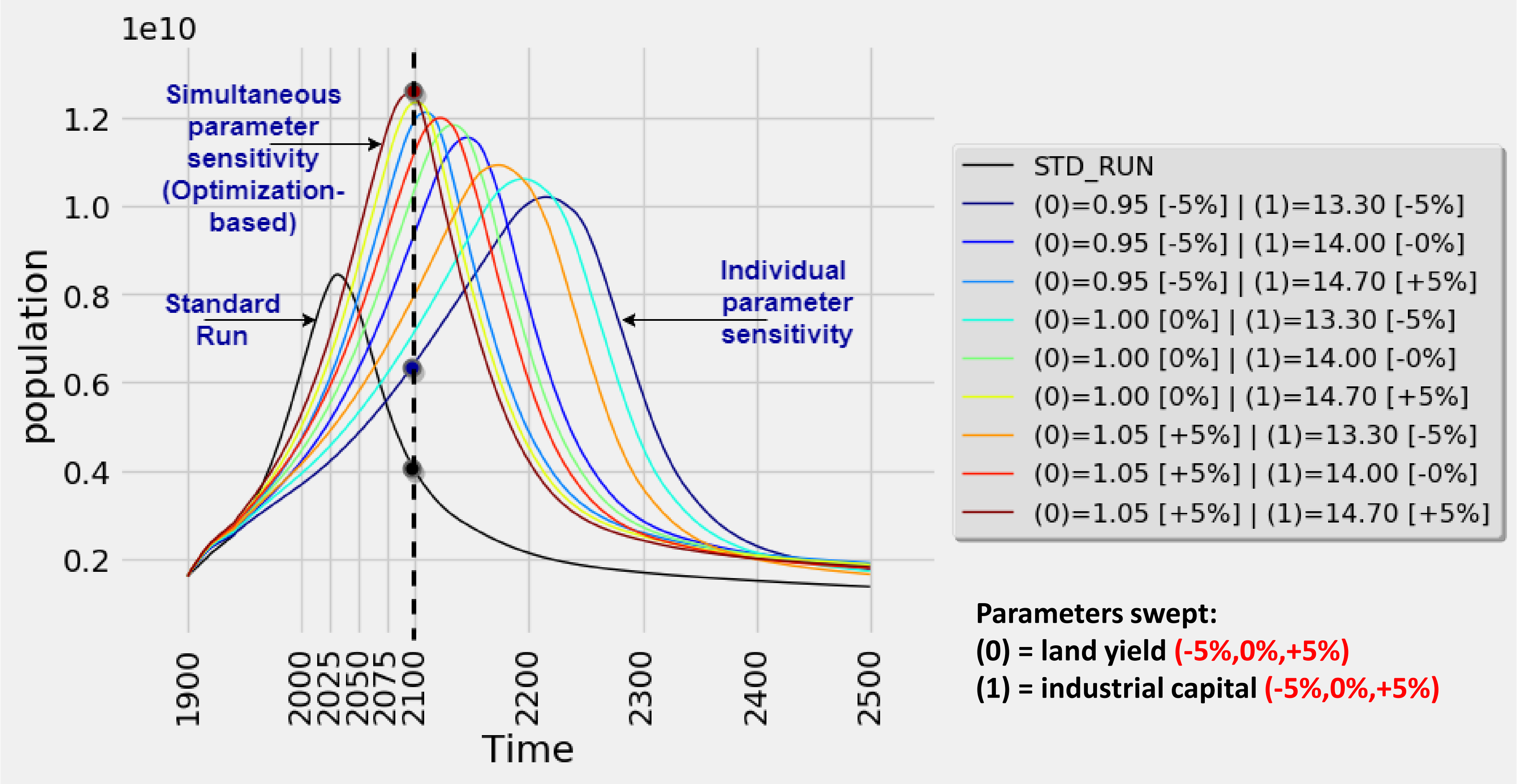
OMSens Simultaneous Param-based Sensitivity Analysis

- To define an experiment:
  - The state **variable** to analyze
  - The set of **parameters** to perturb
  - The allowed **perturbation intervals** for each parameter
- Goal: **pinpoint** small number of **parameters** that **produce** the **largest deviations** when perturbed within small ranges around their default values
- To select parameters and their intervals is not a trivial task
  - Responsibility relies completely on the expertise of the user
  - Enabling all parameters can lead to very costly experiments
- Approach: use a **top-N subset** of parameters from a **ranked list**
  - obtained using **individual parameter-based analysis**

World3 Model



World3 Model



Parameters swept:
(0) = land yield (-5%,0%,+5%)
(1) = industrial capital (-5%,0%,+5%)

Conclusions

- **OMSens** is powerful and flexible tool for **parameter sensitivity analysis** on Modelica models
- **Individual** and **simultaneous** approaches offered varied answers

Future Work

- Experiment with other flavors of the CURVI optimization family. We used CURVIF but CURVIG and CURVIH also available. All globally convergent.