



# PARADOM

*Parallel Algorithmic Differentiation in OpenModelica for Energy-Related Simulations and Optimizations*

Willi Braun, Kshitij Kulshreshtha and Martin Flehmig

OpenModelica Workshop 2017, 06.02.2017

E-Mail: [martin.flehmig@tu-dresden.de](mailto:martin.flehmig@tu-dresden.de)

# PARADOM

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## Change in Generation of Electricity

# In Former Times

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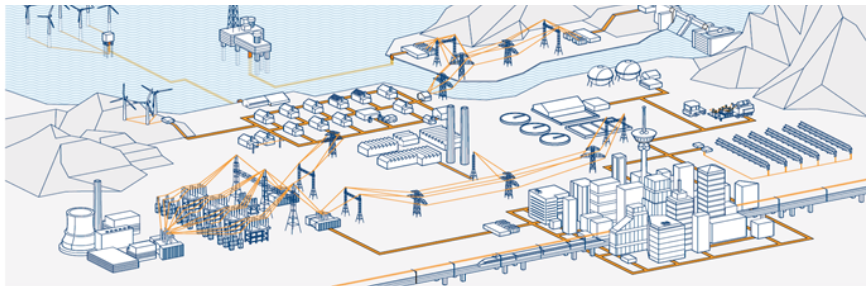
- Classically large power plants with predetermined plans of action



Brown coal power station in Jämschwalde (Germany), [guentherhh](#), CC BY 2.0

# Today and in Future

- Many distributed small power producers (technologies: solar power, wind energy, biogas, etc.)
- Conventional power stations ensure basic demand and peaks



©ABB

## Challenges

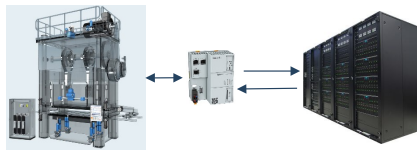
- Combine small producers and conventional power plants into virtual power plants
- Real time optimization of all producers, i.e., flexible adaption to demands

## Optimal Control and Model Predictive Control

# Motivation II

## Optimal Control

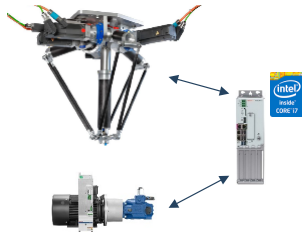
- Measured data of the real process are automatically transferred to the HPC-system
- Controller triggers a dynamic optimization at defined time
- Optimization is executed on HPC-system
- Adaption to changed production conditions



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## Model Predictive Control

- Predictive control basing on a model
- Dynamic optimization problem is solved in every controller cycle
- Real-time requirements
- High-performance multi-core control hardware



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# Round-up of Motivation

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## Tasks

- Modelling of the energy-related facilities and their components
- Simulation and Optimization
  - Components and processes
  - Performance of products in applications
- Online optimization to allow flexible adaption to demands and conditions

## Challenges

- Rapidly increasing systems
- More and more comprehensive and complex models
- Limits of available optimization technologies will be reached in near future



# Computation of Derivatives

---

The considered simulations and optimizations fundamentally base on the efficient computation of first and higher order derivatives.

How to obtain derivatives?

- Hand Coded
  - Implement analytical expression for the derivatives
- Finite Differences
  - Approximation of the derivatives by difference quotients, e.g.,  
$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$
- Symbolic Differentiation
  - Make use of computer algebra systems

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We'll know downsides.

But, can we do better?

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But, can we do better?

**Yes, we can!**

# Computation of Derivatives

---

## Algorithmic Differentiation (AD)

- Computing analytic derivatives of functions present in source code
- Exact derivatives within machine precision
- Low overhead

## Basic idea

- Function present in source code is can be seen as a sequence of elementary arithmetic operations and functions
- Analytic differentiation of elementary functions + propagation by chain rule

Two basic modes: Forward and reverse

# AD forward by Example

---

$$y = f(x_1, x_2, x_3) = \sin(x_1 x_2) x_3$$

- Decompose original function  $f$  into intrinsic functions

$v_1$	=	$x_1 x_2$
$v_2$	=	$\sin(v_1)$
$v_3$	=	$v_2 x_3$
<hr/>		
$y$	=	$v_3$

# AD forward by Example

$$y = f(x_1, x_2, x_3) = \sin(x_1 x_2) x_3$$

- Decompose original function  $f$  into intrinsic functions
- Associate each intermediate variable  $v$  with a derivative  $\dot{v} = \frac{\partial v_i}{\partial x}$
- Apply chain rule

$v_1$	=	$x_1 x_2$
$v_2$	=	$\sin(v_1)$
$v_3$	=	$v_2 x_3$
<hr/>		
$y$	=	$v_3$

$\dot{v}_1$	=	$\dot{x}_1 x_2 + x_1 \dot{x}_2$
$\dot{v}_2$	=	$\cos(v_1) \dot{v}_1$
$\dot{v}_3$	=	$\dot{v}_2 x_3 + v_2 \dot{x}_3$
<hr/>		
$\dot{y}$	=	$\dot{v}_3$

# AD forward by Example

$$y = f(x_1, x_2, x_3) = \sin(x_1 x_2) x_3,$$

What is  $\frac{\partial y}{\partial x_1}$  at (1, 3, 7)?

- Chose  $x_1$  as only independent variable, thus  $\dot{x}_1 = 1$ ,  $\dot{x}_2 = 0$  and  $\dot{x}_3 = 0$

$v_1$	$=$	$x_1 x_2$	$=$	3
$\dot{v}_1$	$=$	$\dot{x}_1 x_2 + x_1 \dot{x}_2$	$=$	3
$v_2$	$=$	$\sin(v_1)$	$=$	0.14112
$\dot{v}_2$	$=$	$\cos(v_1) \dot{v}_1$	$=$	-2.96997
$v_3$	$=$	$v_2 x_3$	$=$	0.98784
$\dot{v}_3$	$=$	$\dot{v}_2 x_3 + v_2 \dot{x}_3$	$=$	-20.78984
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- Derivatives within working accuracy
- All gradients cost  $\mathcal{O}(n)$  function evaluations

# AD in OpenModelica

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So, should we implement AD functionality in OpenModelica?

No, use a well established tool, like ADOL-C!

Package ADOL-C (**A**utomatic **D**ifferentiation by **O**ver**L**oading in **C**++)

- Open-Source
- Used in many applications
- Hugh range of functions
- Bases on operator overloading in C/C++

```
class adouble {  
    double val;  
    double dot;  
}
```

```
adouble operator* (adouble a, adouble b) {  
    adouble c;  
    c.val = a.val * b.val;  
    c.dot = a.dot * b.val + a.val * b.dot;  
    return c;  
}
```

## OpenModelica + ADOL-C

- First prototype 2014 with C++ code and `adouble`
- New prototype generates directly a *trace*
- No compilation needed, just read the *trace*

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Example:

```
model A
  parameter Real a=-0.25;
  Real x,y;
equation
  der(y) = y/x + x*3.0 + a;
  der(x) = x + log(x)*(-3.0);
end A;
```

# Connecting ADOL-C and OpenModelica

## OpenModelica + ADOL-C

- First prototype 2014 with C++ code and `adouble`
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### Example:

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end A;
```

### Trace for model A:

```
// allocation of used variables
{ op:assign_d_zero loc:0 }
{ op:assign_d_zero loc:1 }
{ op:assign_d_zero loc:2 }
{ op:assign_d_zero loc:3 }
// define independent -> x, y
{ op:assign_ind loc:0 }
{ op:assign_ind loc:1 }
// operations
{ op:div_a_a loc:1 loc:0 loc:4 }
{ op:mult_d_a loc:0 loc:5 val:3.0 }
{ op:assign_p loc:1 loc:6 }
{ op:plus_a_a loc:5 loc:6 loc:7 }
{ op:plus_a_a loc:4 loc:7 loc:3 }
{ op:log_op loc:0 loc:4 }
{ op:mult_d_a loc:4 loc:5 val:-3.0 }
{ op:plus_a_a loc:0 loc:5 loc:2 }
// define dependent -> der(x), der(y)
{ op:assign_dep loc:2 }
{ op:assign_dep loc:3 }
// death_not
{ op:death_not loc:0 loc:9 }
// num real parameters
{ op:set_numparam loc:1 }
```

# OpenModelica + ADOL-C - First Results

Example:

ScalableTestSuite.Elementary.SimpleODE.Models.CascadedFirstOrder

## Sparse Jacobian Evaluation:

N	ADOL-C	OM Symbolical
100	0.000480442	0.000156783
200	0.000830835	0.000413299
400	0.00157551	0.000952923
800	0.00294508	0.00209405
1600	0.00676732	0.00536921
3200	0.0141433	0.012003
6400	0.0390204	0.0310391
12800	0.0771545	0.0756394

## Generate and Read Performance:

N	ADOL-C		OM Sym.
	generate	read	generate
100	0.0008721	0.0289017	0.0255
200	0.001601	0.0569519	0.04937
400	0.004216	0.114088	0.1044
800	0.006973	0.227438	0.2311
1600	0.01347	0.521812	0.557
3200	0.0259	0.898992	1.732
6400	0.05123	1.8135	4.436
12800	0.1087	3.62892	43.57

**Status:** *Early-pre-alpha* prototype

We can create traces for (simple) expressions using

- standard operators (e.g., +, -, \*, /)
- and standard functions (e.g., sin, cos, log, exp).

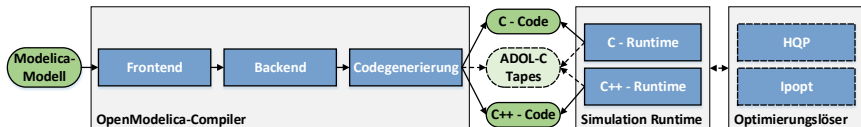
Outlook

- If expressions
- Arrays
- Records, functions, algorithms
- Algebraic loops

# Goals within PARADOM

Development of parallel algorithms and application of efficient optimization methods within the OpenModelica environment with respect to HPC systems

- Integration of ADOL-C into the OpenModelica Compiler and the simulation runtimes C and C++
- Linking of the optimization solvers Ipopt and HQP and OpenModelica
- Provisioning of interfaces to suitable solvers for systems of equations (e.g., MUMPS, SuperLU, SuiteSparse)
- Parallelization of derivative computation in ADOL-C
- Development of parallel multiple shooting methods within HQP





# Wide Appeal and Sustainability

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OpenModelica, ADOL-C and HQP are open-source software projects

- We will develop on the corresponding repositories
- Developments can be used immediately by the communities
- Feedback from the users
- Early and continuous build-up of know-how on user side

ADOL-C: parallel computation of derivatives

- Independent from OpenModelica
- Guarantees future

OpenModelica

- Enable and speed-up large simulations
- New users due to new possibilities/capabilities

Thank you for your attention.