

PlanarMechanics v1.2

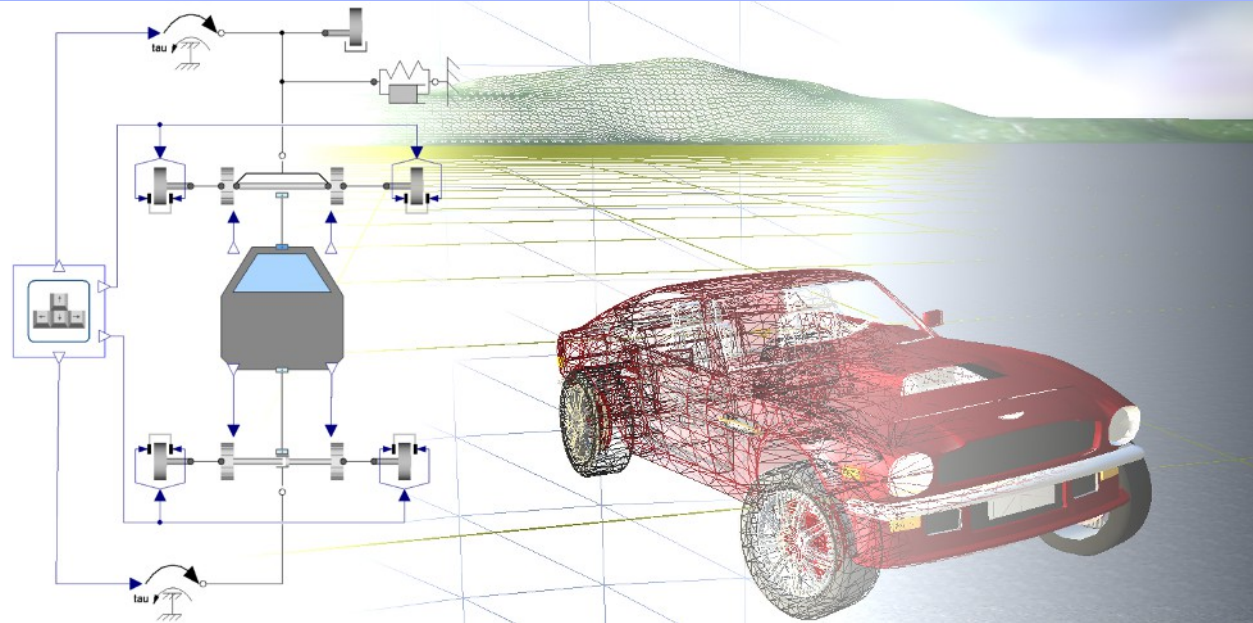
A free Modelica Library

OpenModelica Annual Workshop,
February 3, 2014,
Linköping, Sweden

equation

```
sx0 = cos(frame_a.phi)*sx_norm + ...  
sy0 = -sin(frame_a.phi)*sx_norm + ...  
vy = der(frame_a.y);  
w_roll = der(flange_a.phi);  
v_long = vx*sx0 + vy*sy0;  
v_lat = -vx*sy0 + vy*sx0;  
v_slip_lat = v_lat - 0;  
v_slip_long = v_long - R*w_roll;
```

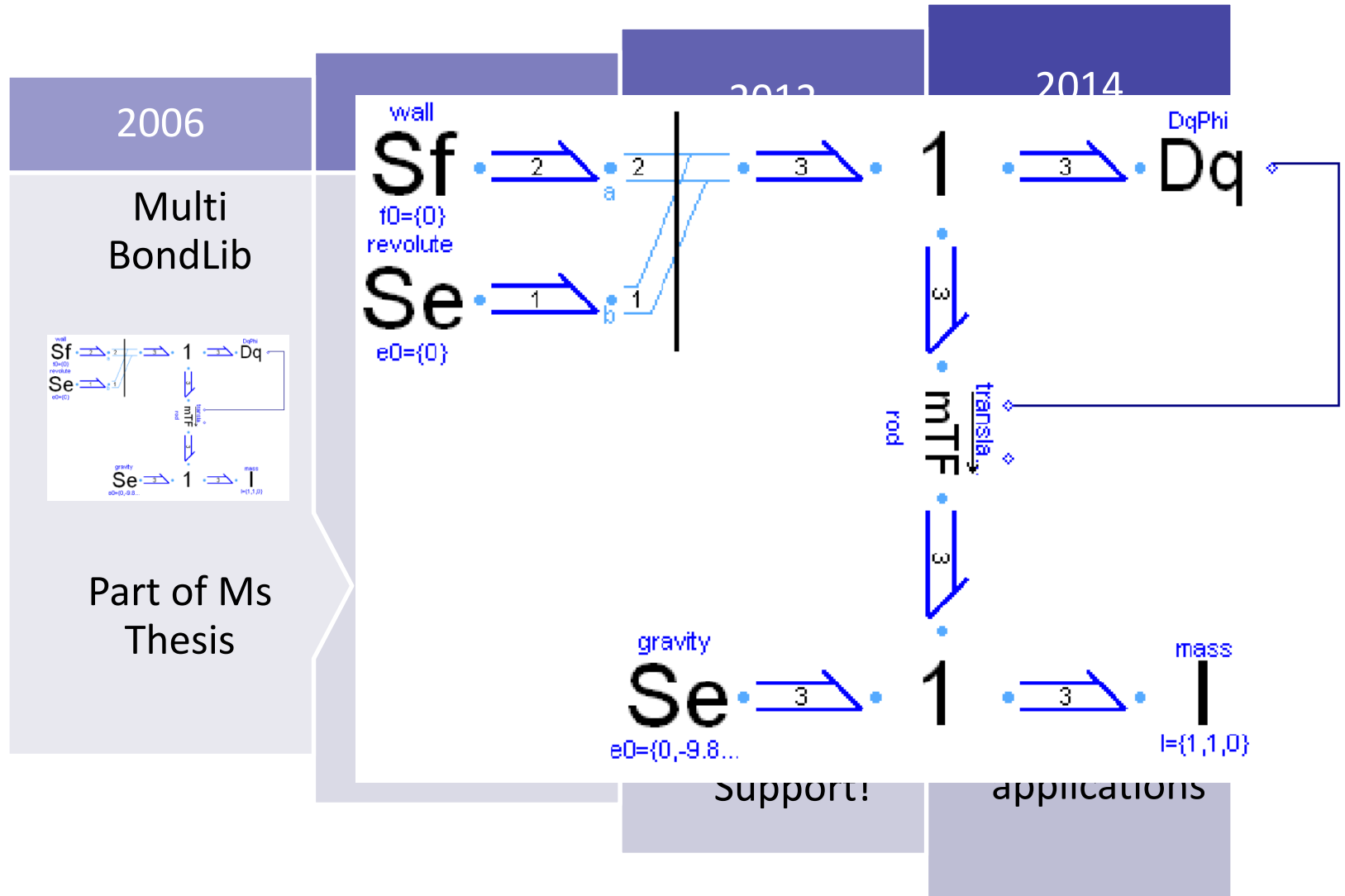
```
v_slip = sqrt(v_slip_long^2 + ...  
-f_long*R = flange_a.tau;  
frame_a.t = 0;  
f = N*. S_Func(vAdhesion,vSlide,...  
f_long =f*v_slip_long/v_slip;  
f_lat =f*v_slip_lat/v_slip;  
f_long = frame_a.fx*sx0 + ...  
f_lat = -frame_a.fx*sy0 + ...
```

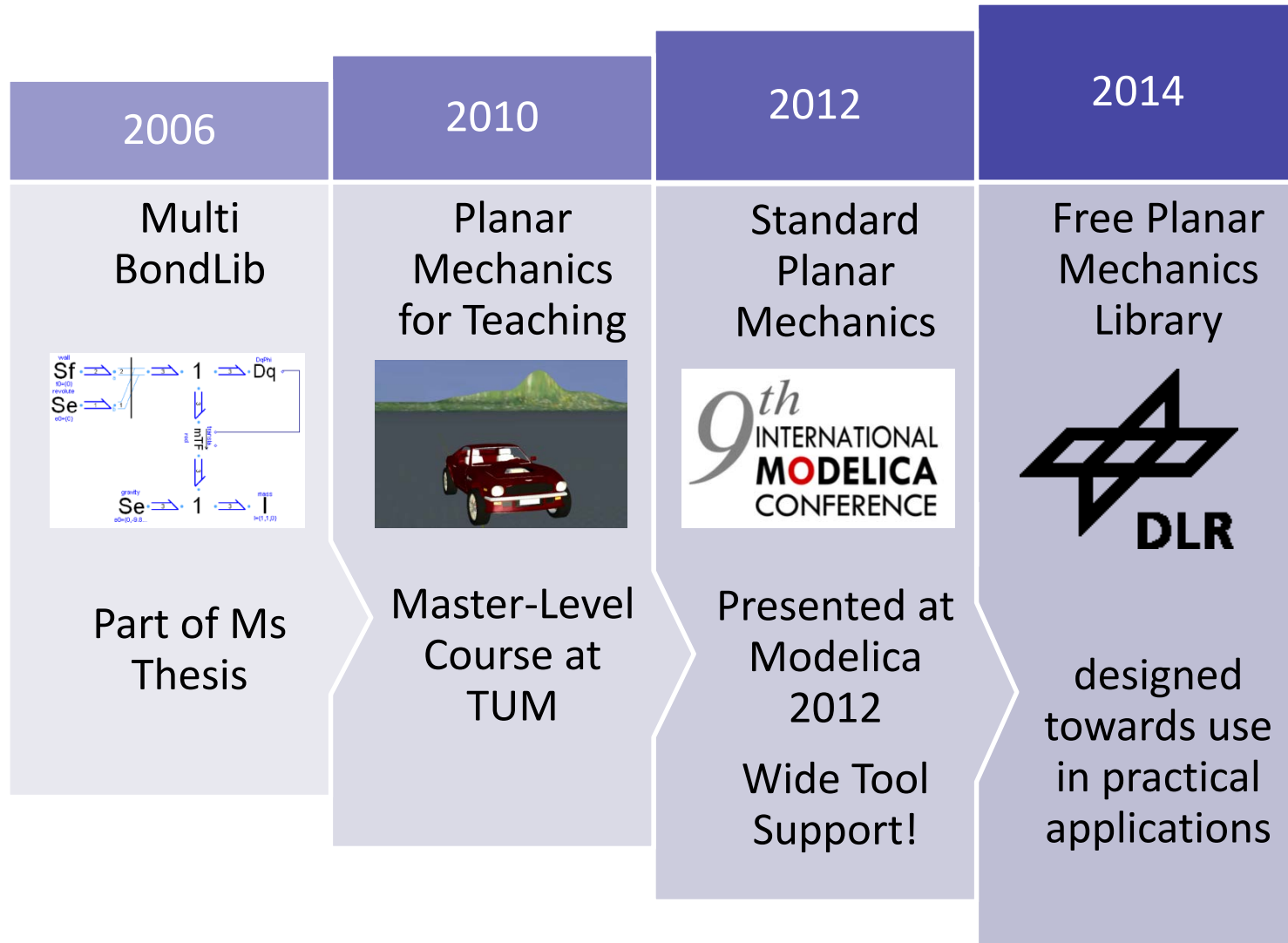


Dr. Dirk Zimmer, Franciscus van der Linden, Zheng Qu

German Aerospace Center (DLR), Institute of System Dynamics and Control

Timeline





Dirk Zimmer (DLR)

- initial version, wheel models, etc.

Franciscus van der Linden (DLR)

- Gearbox components
- spring and dampers

Zhen Qu (DLR, TUM)

- Sensors,
- World model,
- improved animation, initialization, documentation

Part 1:

- Basics of Planar Mechanics (Excerpts from my lecture)

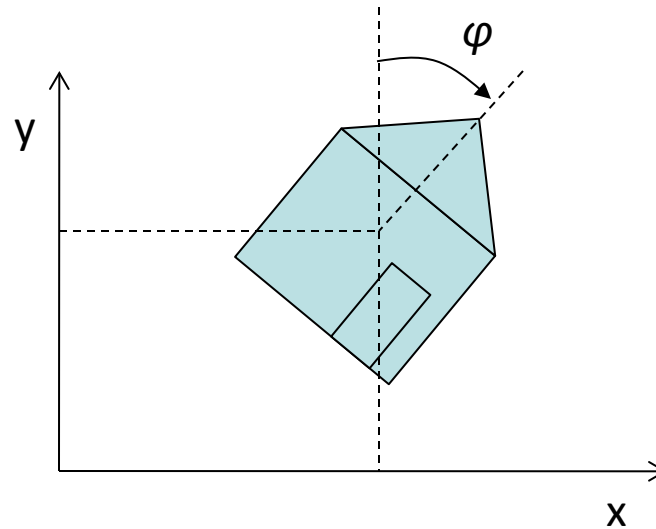
Part 2:

- New features / improvements of the free PlanarMechanics Library

Part 1:

- Basics of Planar Mechanics (Excerpts from my lecture)

- In planar mechanics, we describe the physics of a multi body system in a two-dimensional plane.



- In the planar world, all motions and positions can be described by two translational positions and an angular orientation
- By convention we denote the horizontal position with x , the vertical position with y and the orientation by the angle φ (phi).

Why Planar Mechanics?

- **Essentially: Sometimes 1D is too simple, 3D is too complex.**
- Tangible and visual systems
- The simulation results can be visualized and animated.
- Fundamental formulas (Newton's law, D'Alembert's principle) are taught already in high-school.
- Everyone has an intuitive understanding about the motion of mechanical systems and how to control it.
- Interactions to most other domains. (Electrical Engines, Hydraulics, Heat)

- From 1D-mechanics, we know that we should choose force and torque as flow-variables and position and angle as potential variables.
- Planar mechanics combine three 1D-subsystems. Hence the following connector design seems natural.

Potential variables

x (horizontal position)

y (vertical position)

φ (orientation angle)

Flow variables

f_x (horizontal force)

f_y (vertical force)

τ (torque)

- Here, the corresponding Modelica-Code:

```
connector Frame
  "General Connector for planar mechanical components"

  SI.Position x      "x-position";
  SI.Position y      "y-position";
  SI.Angle phi       "angle (clockwise)";
  flow SI.Force fx   "force in x-direction";
  flow SI.Force fy   "force in y-direction";
  flow SI.Torque t   "torque (clockwise)";

end Frame;
```

- It is common style to extend two connectors with different icons from the general connector.
- Some components contain characteristics that are directed. Hence it is helpful to see, if your connecting to side A or side B.

```
connector Frame_a  
  extends Frame;  
end Frame_a;
```

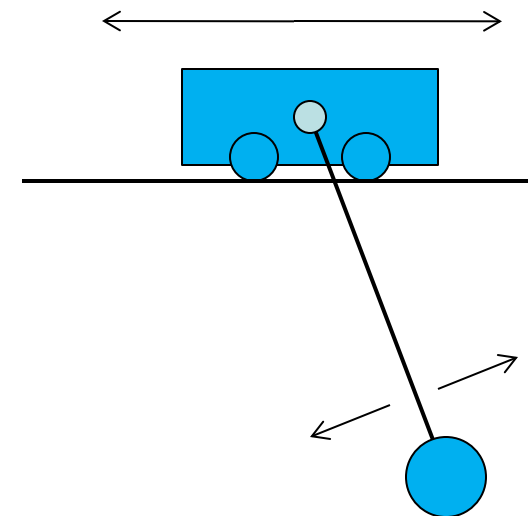


```
connector Frame_b  
  extends Frame;  
end Frame_b;
```



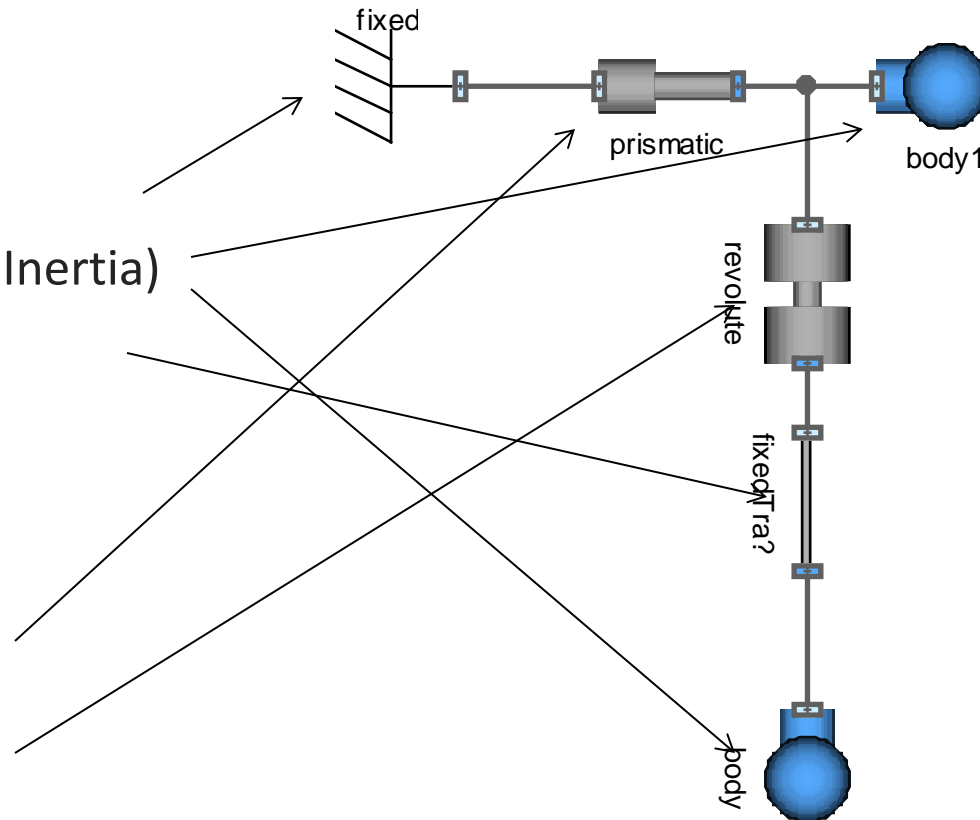
- All of these connectors are collected in an interface package.

- Using this connector, we can build all components for a crane crab
- Parts
 - Wall
 - Body (Mass and Inertia)
 - FixedTranslation
 - FixedRotation
- Joints
 - Prismatic Joint
 - Revolute Joint

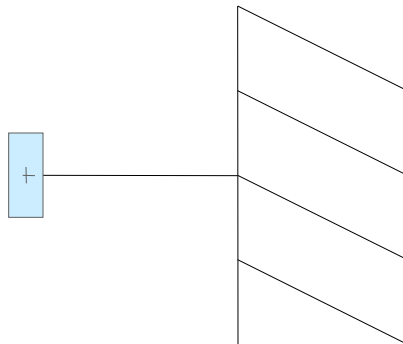


Decomposition into components

- Parts
 - Wall
 - Body (Mass and Inertia)
 - FixedTranslation
 - FixedRotation
- Joints
 - Prismatic Joint
 - Revolute Joint

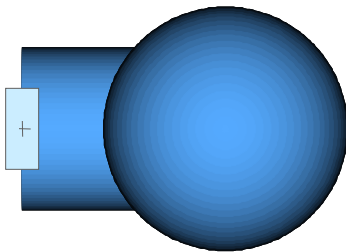


We can already model the first basic components. Let us start with the wall component that represents a fixation point.



```
model Fixed "FixedPosition"  
  
    Interfaces.Frame_a frame_a;  
    parameter SI.Position r[2] = {0,0};  
        "fixed x-position";  
    parameter SI.Angle phi = 0  
        "fixed angle";  
  
    equation  
        {frame_a.x, frame_a.y} = r;  
        frame_a.phi = phi;  
  
end Fixed;
```

- A little more elaborate is the body-component that represents a mass with inertia.



- Essentially, the model formulates Newton's law.

```
model Body
  Interfaces.Frame_a frame_a;

  parameter SI.Mass m;
  parameter SI.Inertia I;

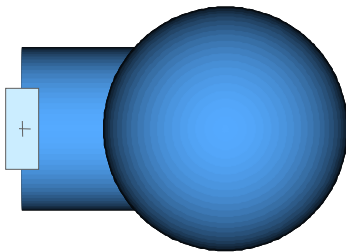
  SI.Force f[2];
  SI.Position r[2];
  SI.Velocity v[2];
  SI.Acceleration a[2];
  SI.AngularVelocity w;
  SI.AngularAcceleration z;

equation
  r = {frame_a.x, frame_a.y};
  v = der(r);
  w = der(frame_a.phi);

  a = der(v);
  z = der(w);

  f = {frame_a.fx, frame_a.fy};
  f = m*a;
  frame_a.t = I*z;
end Body
```

- Since the gravitational force is dependent on the mass ($m \cdot g$), it makes sense to compute right in the body model.



- A parameter for the gravitational acceleration is added and Newton's law is extended.

```
model Body
  Interfaces.Frame_a frame_a;

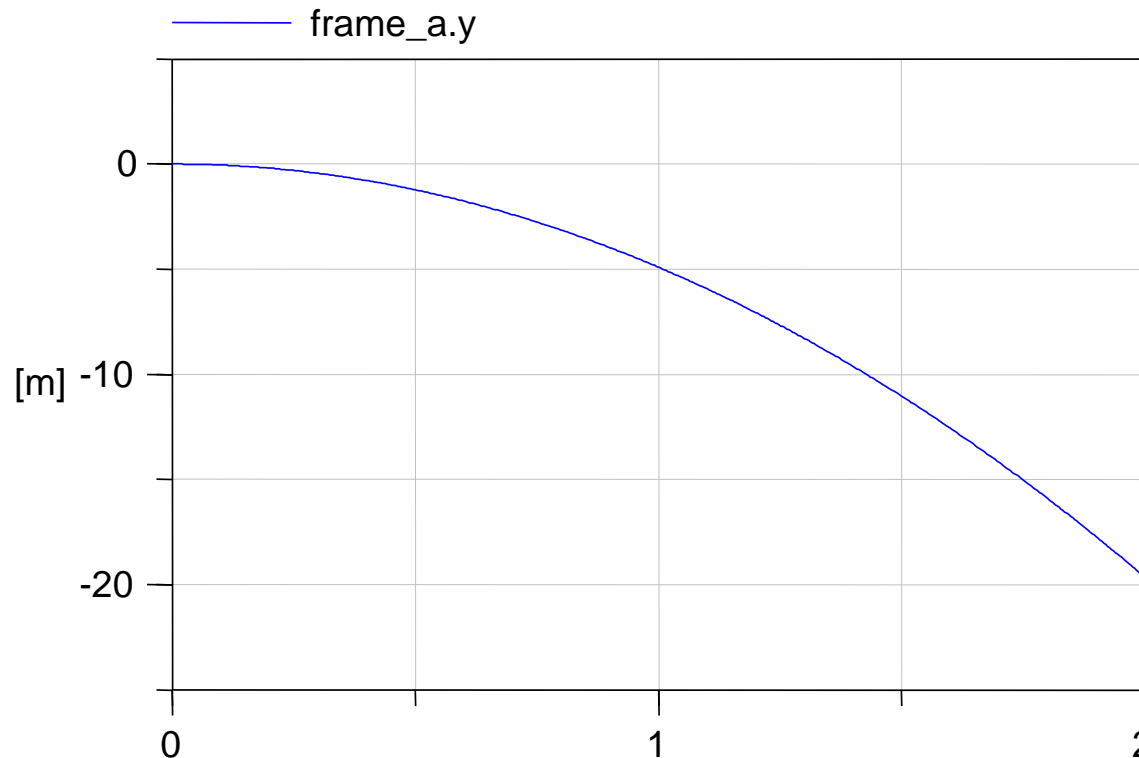
  parameter SI.Mass m;
  parameter SI.Inertia I;
  parameter SI.Acceleration[2] g={0,-9.81};
  SI.Force f[2];
  SI.Position r[2];
  SI.Velocity v[2];
  SI.Acceleration a[2];
  SI.AngularVelocity w;
  SI.AngularAcceleration z;

equation
  r = {frame_a.x, frame_a.y}
  v = der(r);
  w = der(frame_a.phi);

  a = der(v);
  z = der(w);

  f = {frame_a.fx, frame_a.fy};
  f + m*g = m*a;
  frame_a.t = I*z;
end Body
```


- Here is the simulation result:



- It shows the parabolic descent of a body due to gravity acceleration.

- Components that have two frames are little more difficult.
- Let us start by modeling a neutral component.
- The model itself is rather meaningless but it represents a good starting point for the design of any new component.

```
model Neutral
  Interfaces.Frame_a frame_a;
  Interfaces.Frame_a frame_a;
```

```
equation
```

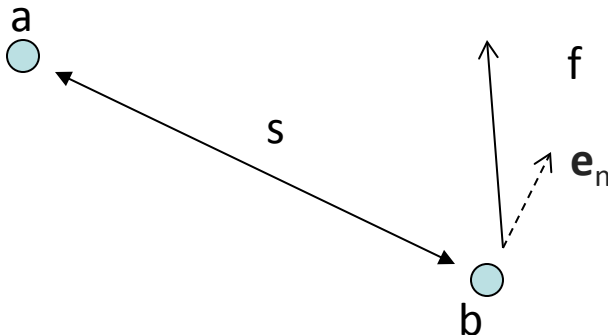
```
frame_a.fx = 0;
frame_a.fy = 0;
frame_a.t = 0;
```

```
frame_a.fx + frame_b.fx = 0;
frame_a.fy + frame_b.fy = 0;
frame_a.t
+ frame_b.t
- (frame_b.x - frame_a.x)*frame_b.fy
+ (frame_b.y - frame_a.y)*frame_b.fx
= 0;
```

```
end Neutral
```

Components with two Flanges

- The model imposes no constraints on the positions.
- This component has two frames, but exhibits no effect.
- The balance equations for the forces contains the lever principle.



$$\tau = \mathbf{f} \cdot \mathbf{e}_n \cdot s = \mathbf{f} \cdot (\mathbf{e}_n \cdot \mathbf{s})$$
$$\tau = (f_x, f_y) \cdot (s_y, -s_x)$$

```
model Neutral
```

```
  Interfaces.Frame_a frame_a;
```

```
  Interfaces.Frame_a frame_a;
```

```
equation
```

```
  frame_a.fx = 0;
```

```
  frame_a.fy = 0;
```

```
  frame_a.t = 0;
```

```
  frame_a.fx + frame_b.fx = 0;
```

```
  frame_a.fy + frame_b.fy = 0;
```

```
  frame_a.t
```

```
  + frame_b.t
```

```
  - (frame_b.x - frame_a.x)*frame_b.fy
```

```
  + (frame_b.y - frame_a.y)*frame_b.fx
```

```
  = 0;
```

```
end Neutral
```

Guidelines:

- For each positional constraint we add, we have to remove the corresponding force equation.
- For each variable that we add, we have to add an equation
- Finally, we may be able to simplify the balance equations.

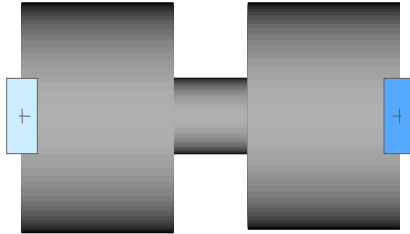
```
model Revolute
  Interfaces.Frame_a frame_a;
  Interfaces.Frame_a frame_a;

equation

  frame_a.fx = 0;
  frame_a.fy = 0;
  frame_a.t = 0;

  frame_a.fx + frame_b.fx = 0;
  frame_a.fy + frame_b.fy = 0;
  frame_a.t
  + frame_b.t
  - (frame_b.x - frame_a.x)*frame_b.fy
  + (frame_b.y - frame_a.y)*frame_b.fx
  = 0;
end Revolute
```

Let us start with the revolute joint:



```
model Revolute
```

```
  Interfaces.Frame_a frame_a;
```

```
  Interfaces.Frame_a frame_b;
```

```
equation
```

```
  frame_a.fx = 0;
```

```
  frame_a.fy = 0;
```

```
  frame_a.t = 0;
```

```
  frame_a.fx + frame_b.fx = 0;
```

```
  frame_a.fy + frame_b.fy = 0;
```

```
  frame_a.t
```

```
  + frame_b.t
```

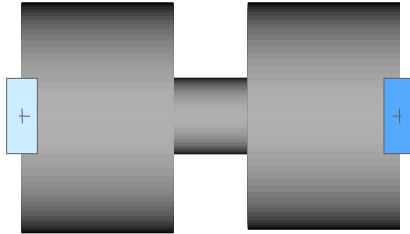
```
  - (frame_b.x - frame_a.x)*frame_b.fy
```

```
  + (frame_b.y - frame_a.y)*frame_b.fx
```

```
  = 0;
```

```
end Revolute
```

Let us start with the revolute joint:



- The translational positions of a and b are equal. (2 constraints)
- No torque can act on the joint.

```
model Revolute
```

```
  Interfaces.Frame_a frame_a;
```

```
  Interfaces.Frame_a frame_b;
```

```
equation
```

```
  frame_a.fx = 0 replaced by
```

```
  frame_a.x = frame_b.x;
```

```
  frame_a.fy = 0 replaced by
```

```
  frame_a.y = frame_b.y;
```

```
  frame_a.t = 0;
```

```
  frame_a.fx + frame_b.fx = 0;
```

```
  frame_a.fy + frame_b.fy = 0;
```

```
  frame_a.t
```

```
  + frame_b.t
```

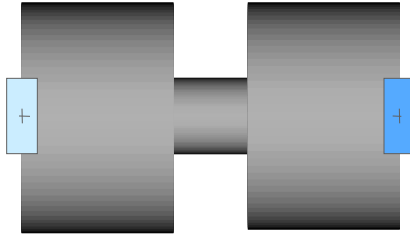
```
  - (frame_b.x - frame_a.x)*frame_b.fy
```

```
  + (frame_b.y - frame_a.y)*frame_b.fx
```

```
  = 0;
```

```
end Revolute
```

Let us start with the revolute joint:



- The translational positions of a and b are equal. (2 constraints)
- No torque can act on the joint.
- The lever principle is redundant here...
- That's it! ...actually

```
model Revolute
  Interfaces.Frame_a frame_a;
  Interfaces.Frame_a frame_b;
```

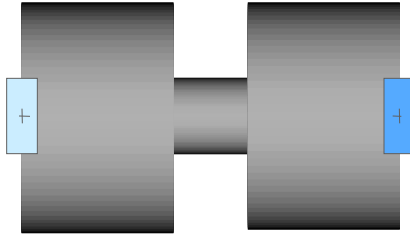
equation

```
frame_a.fx = 0 replaced by
frame_a.x = frame_b.x;
frame_a.fy = 0 replaced by
frame_a.y = frame_b.y;
frame_a.t = 0;
```

```
frame_a.fx + frame_b.fx = 0;
frame_a.fy + frame_b.fy = 0;
frame_a.t + frame_b.t = 0;
```

```
end Revolute
```

Let us start with the revolute joint:



- For completeness, we'd like to add two differential equations for the angle, the angular velocity and its acceleration.
- After all, these variables are of interest.
- We can now use the joint in order to express motion.
- It also helps with initialization.

```
model Revolute
  Interfaces.Frame_a frame_a;
  Interfaces.Frame_a frame_b;

  SI.Angle phi
  SI.AngularVelocity w;
  SI.AngularAcceleration z;

equation
  frame_a.phi + phi = frame_b.phi;
  w = der(phi);
  z = der(w);

  frame_a.x = frame_b.x;
  frame_a.y = frame_b.y;
  frame_a.t = 0;

  frame_a.fx + frame_b.fx = 0;
  frame_a.fy + frame_b.fy = 0;
  frame_a.t + frame_b.t = 0;

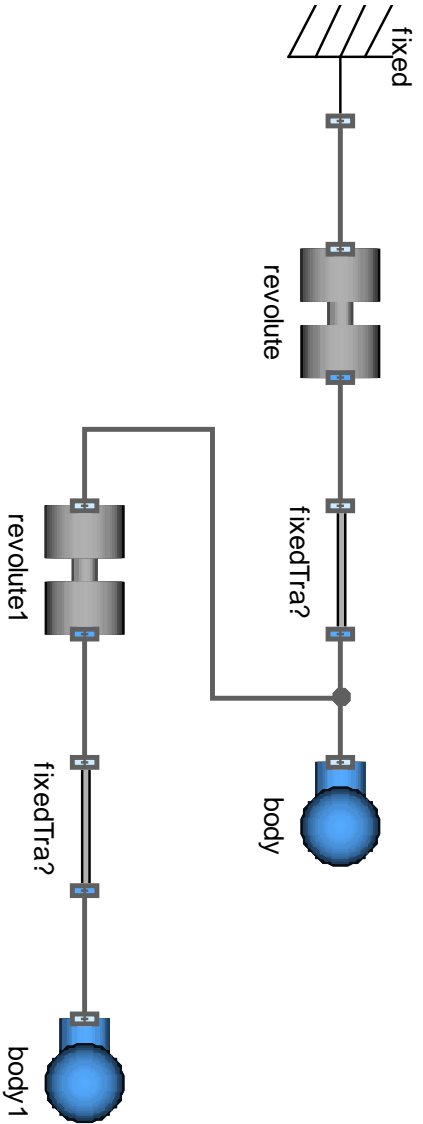
end Revolute
```


- We already have 3 components:
 - Fixation
 - Body
 - Revolute

- The lecture goes on explaining two more components:
 - Fixed Translation
 - Prismatic Joint

- Having just these five components available, we can already assemble many interesting systems.

Double Pendulum



Double Pendulum: Peak Motion

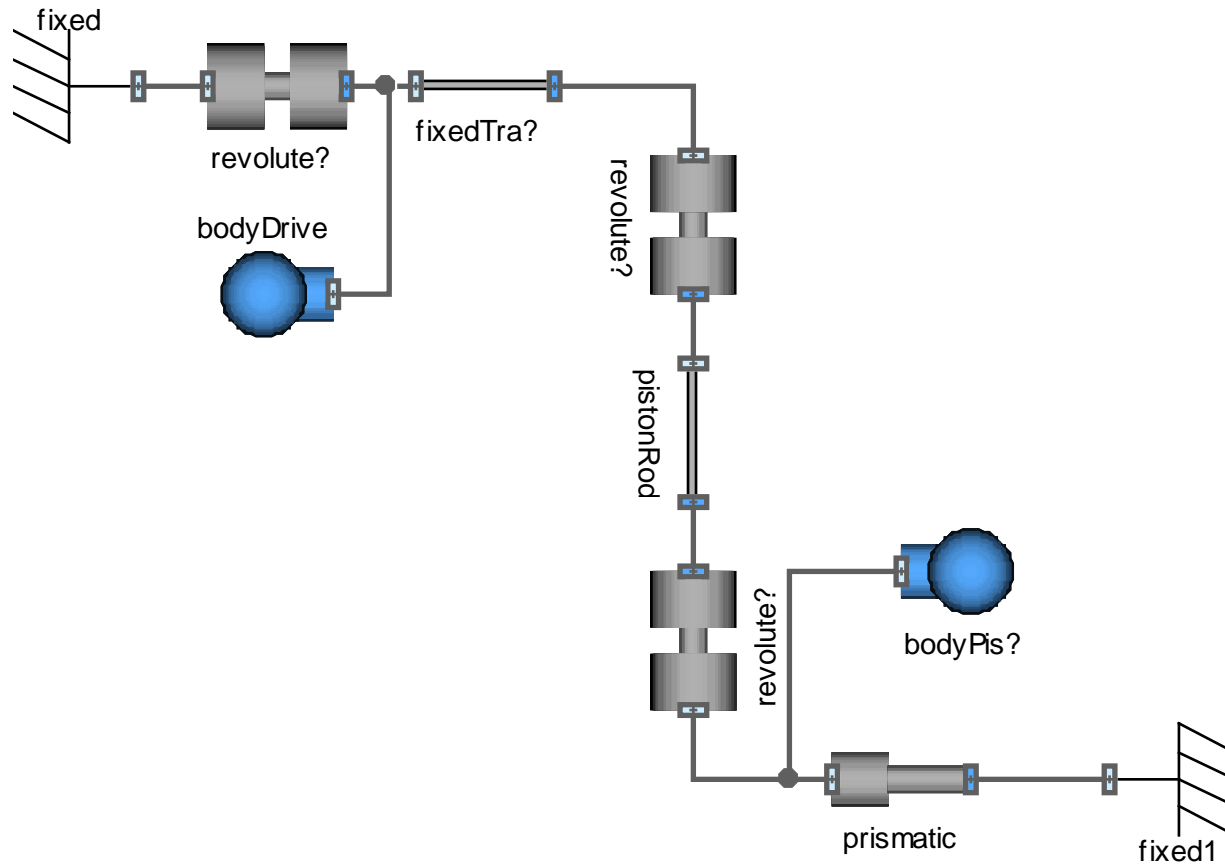


Institute of System Dynamics and Control



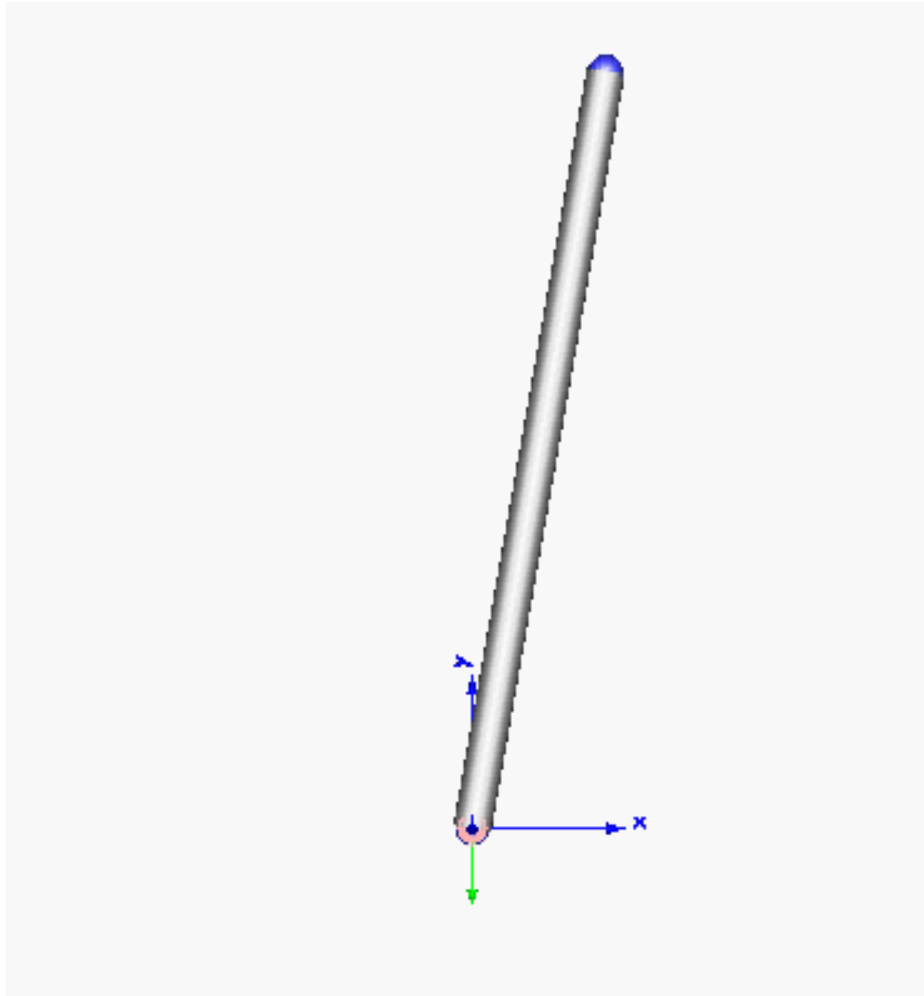
- The double pendulum is wonderful example of a chaotic system.
- Although the individual components are simple, the motion of the system cannot be predicted.

Kinematic Loops

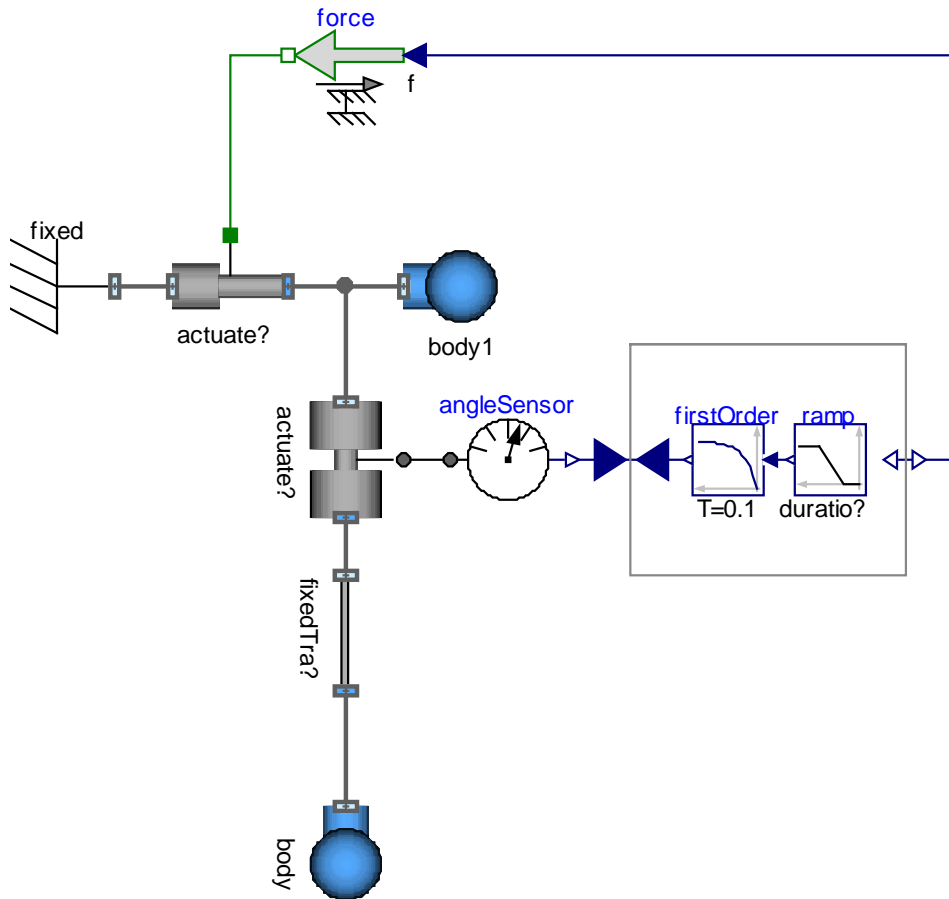




- Discussing kinematic loops we can address the difficulties of initialization
- Also the matter of state-selection can be addressed and it is a good occasion to explain the Pantelides Algorithm and its limitations.

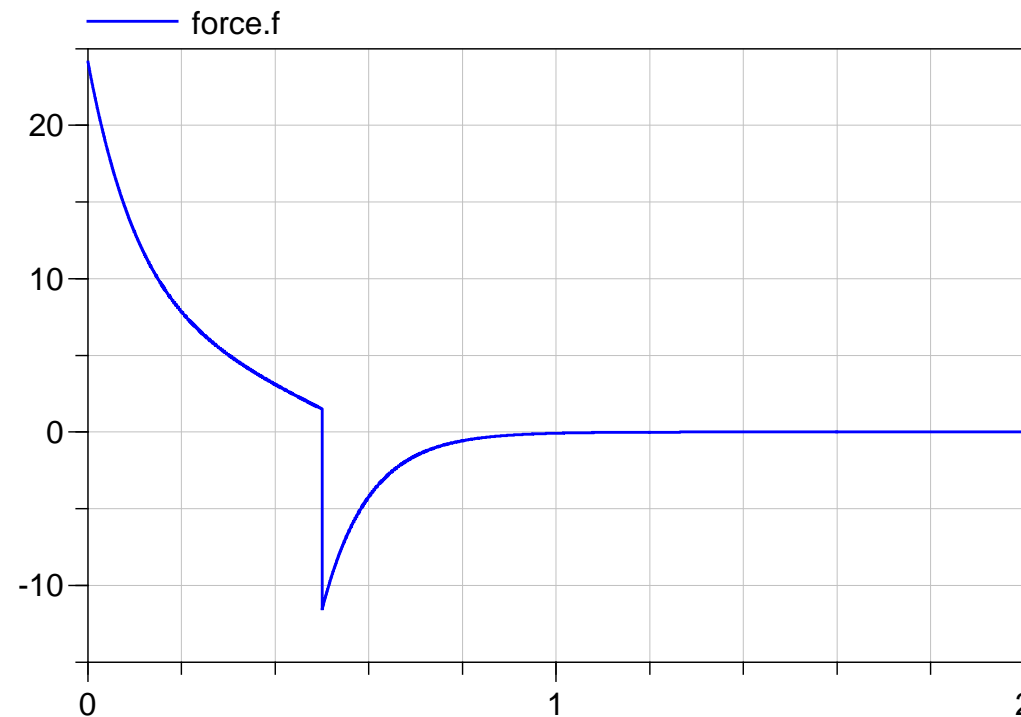


- The task is to balance a pendulum in upright position.
- On the left you see the simulation result using a PD controller for control.



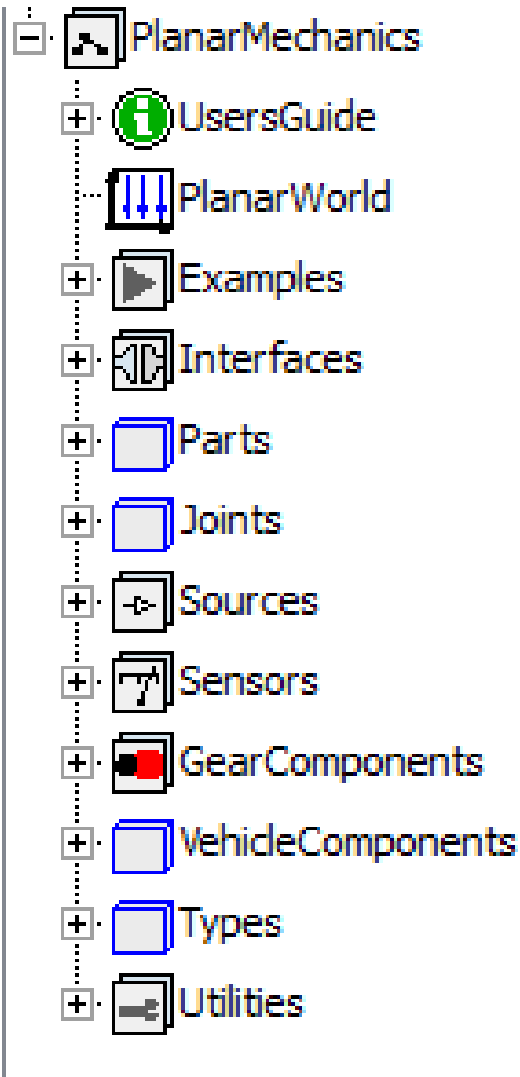
- This is the model of an inverted (upright) pendulum.
- Here, model inversion is applied.
- This means we prescribe the motion and compute the required force
- The motion must be differentiable since this is a higher index system

- The Inverse Pendulum is nice example for control tasks.
- Demonstrating model-inversion reveals to true generality of DAE-based modeling of physical systems.



Part 2:

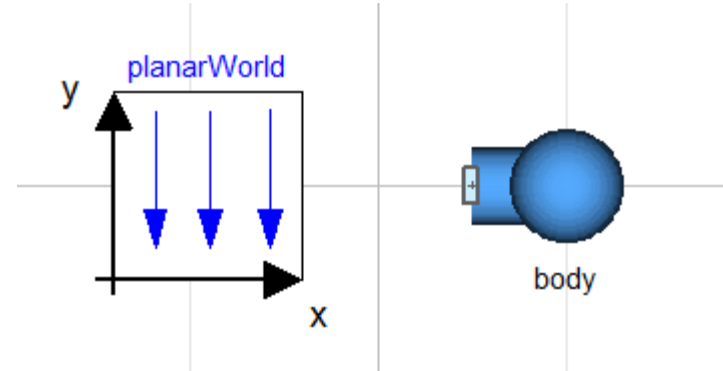
- New features / improvements of the free PlanarMechanics Library



- The library is designed in resemblance to the Multibody library
- There are the classic elements such as
 - Rigid parts
 - Joint elements
 - Sensors
- In addition to these elements, there are simple wheel and tire models and gear components



- Gravity
- Coordinate System
- Defaults for animation



General Animation Defaults

Component

Name


Comment

Model

Path PlanarMechanics.PlanarWorld

Comment

Parameters

enableAnimation	<input type="text"/>	true ▾ ▶
animateWorld	<input type="text"/>	true ▾ ▶
animateGravity	<input type="text"/>	true ▾ ▶
label1	<input type="text"/>	"x" ▶
label2	<input type="text"/>	"y" ▶
g	<input type="text"/>	{0,-9.81}  m/s ²

+ Planar world model

+ Improved initialization

Initialization

phi.start	<input type="checkbox"/>	<input type="text" value="0"/>	deg	Angular position
w.start	<input type="checkbox"/>	<input type="text" value="0"/>	rad/s	Angular velocity
z.start	<input type="checkbox"/>	<input type="text" value="0"/>	rad/s ²	Angular acceleration

General Advanced Animation

stateSelect StateSelect.always | ▾ Priority to use phi and w as states



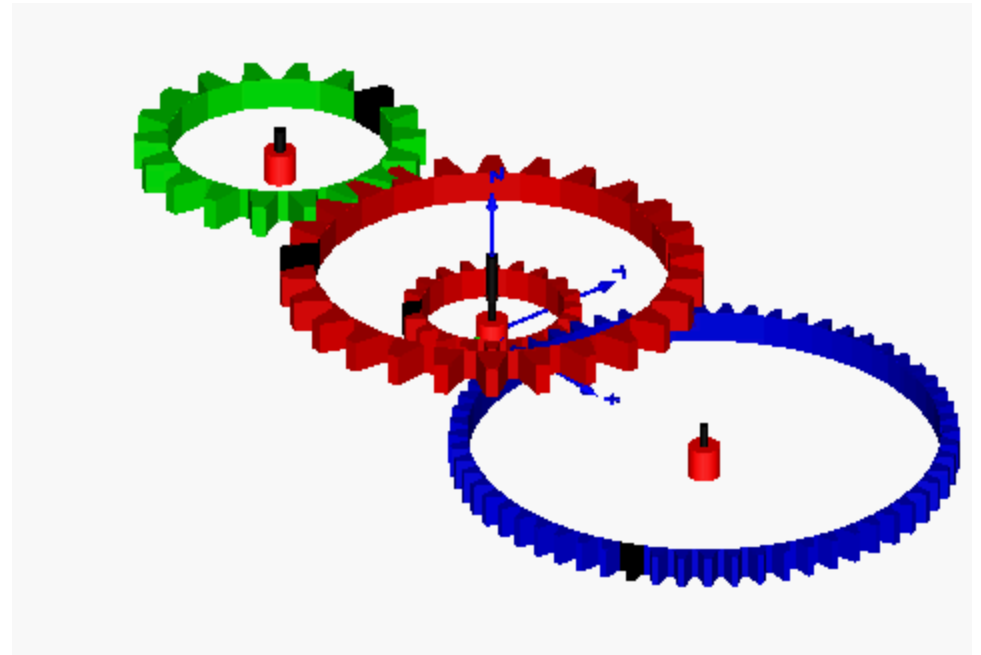
**Planar world
model**



**Improved
initialization**



**Improved
animation
with z-levels**

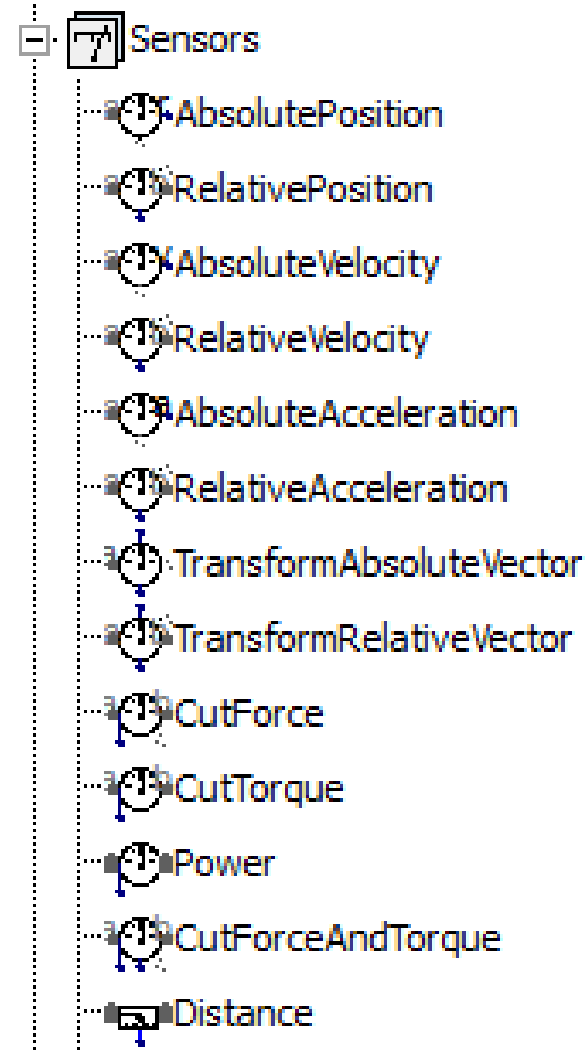


+ Planar world model

+ Improved initialization

+ Improved animation with z-levels

+ Set of sensors



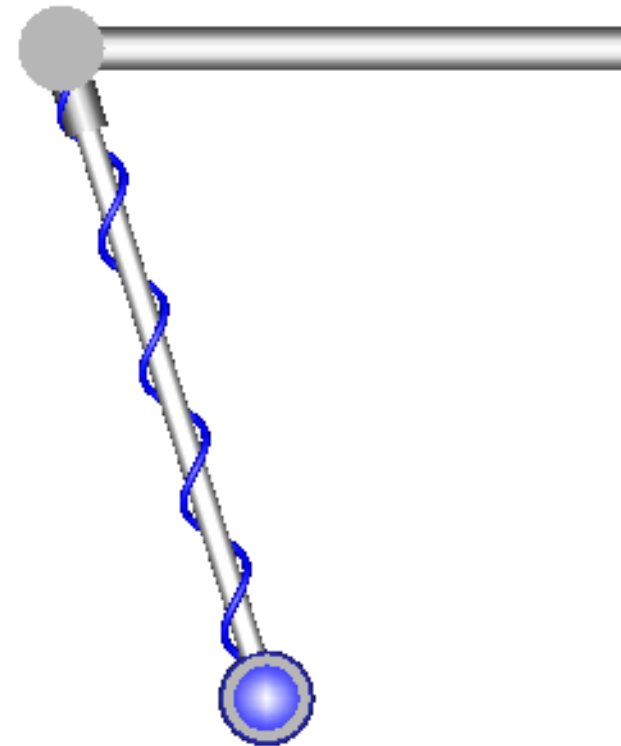
+ Planar world model

+ Improved initialization

+ Improved animation with z-levels

+ Set of sensors

+ Spring and Damper components

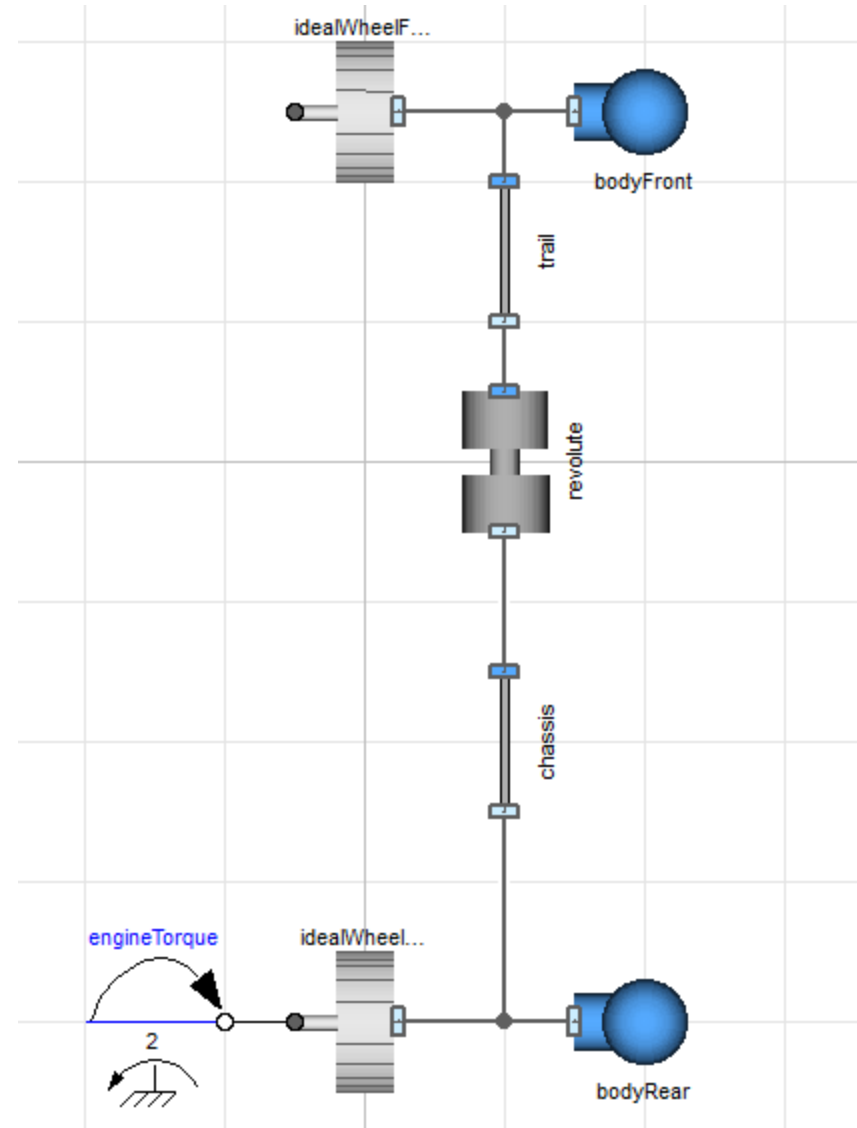


Wheel Joints for

- ideal rolling
- dry-friction based rolling
- slip based rolling

Examples:

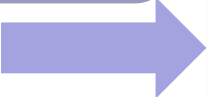
- Single track model
- Two track model

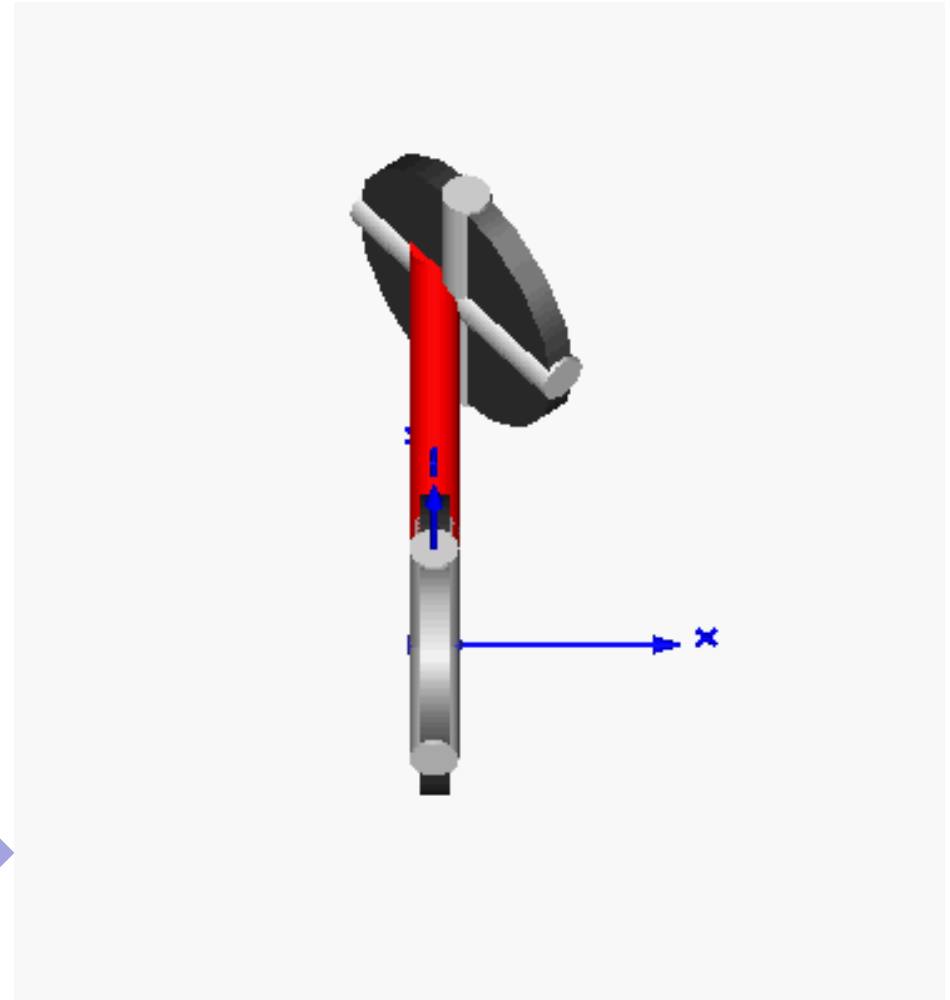


Wheel Joints for

- ideal rolling
- dry-friction based rolling
- slip based rolling

Examples:

- Single track model 
- Two track model

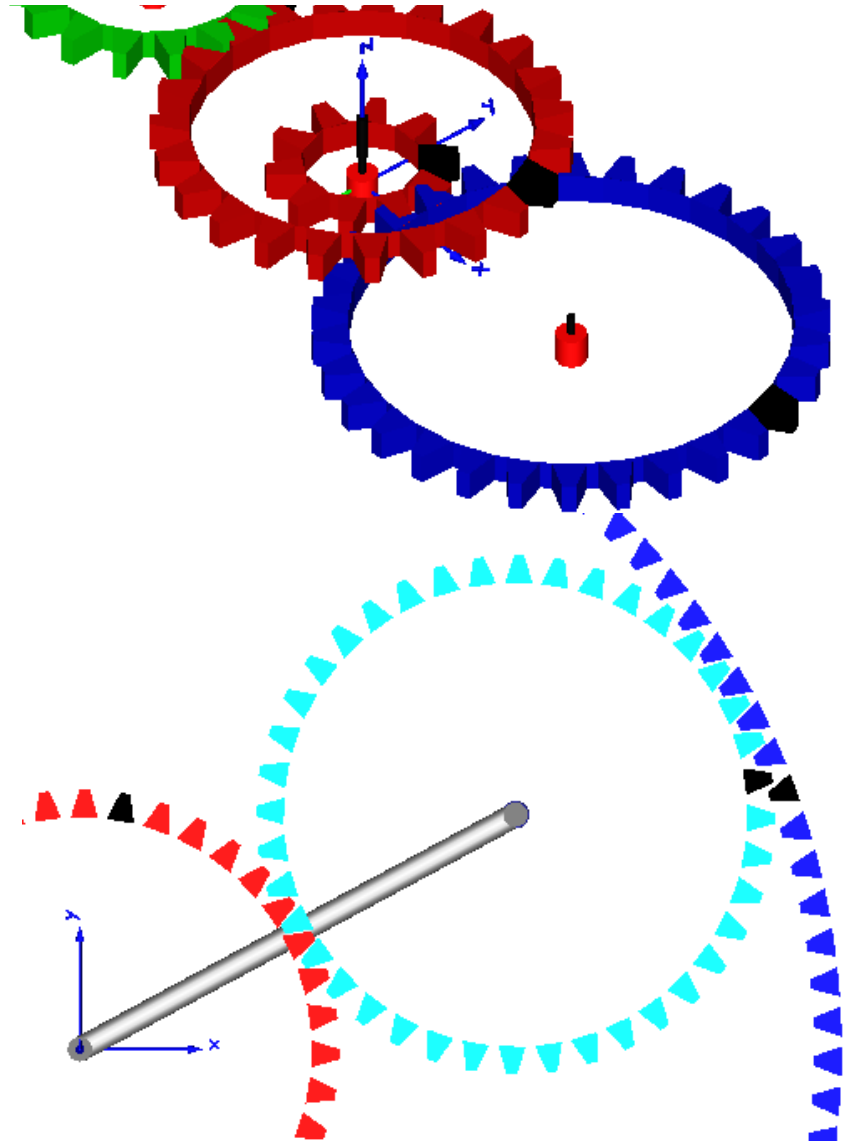


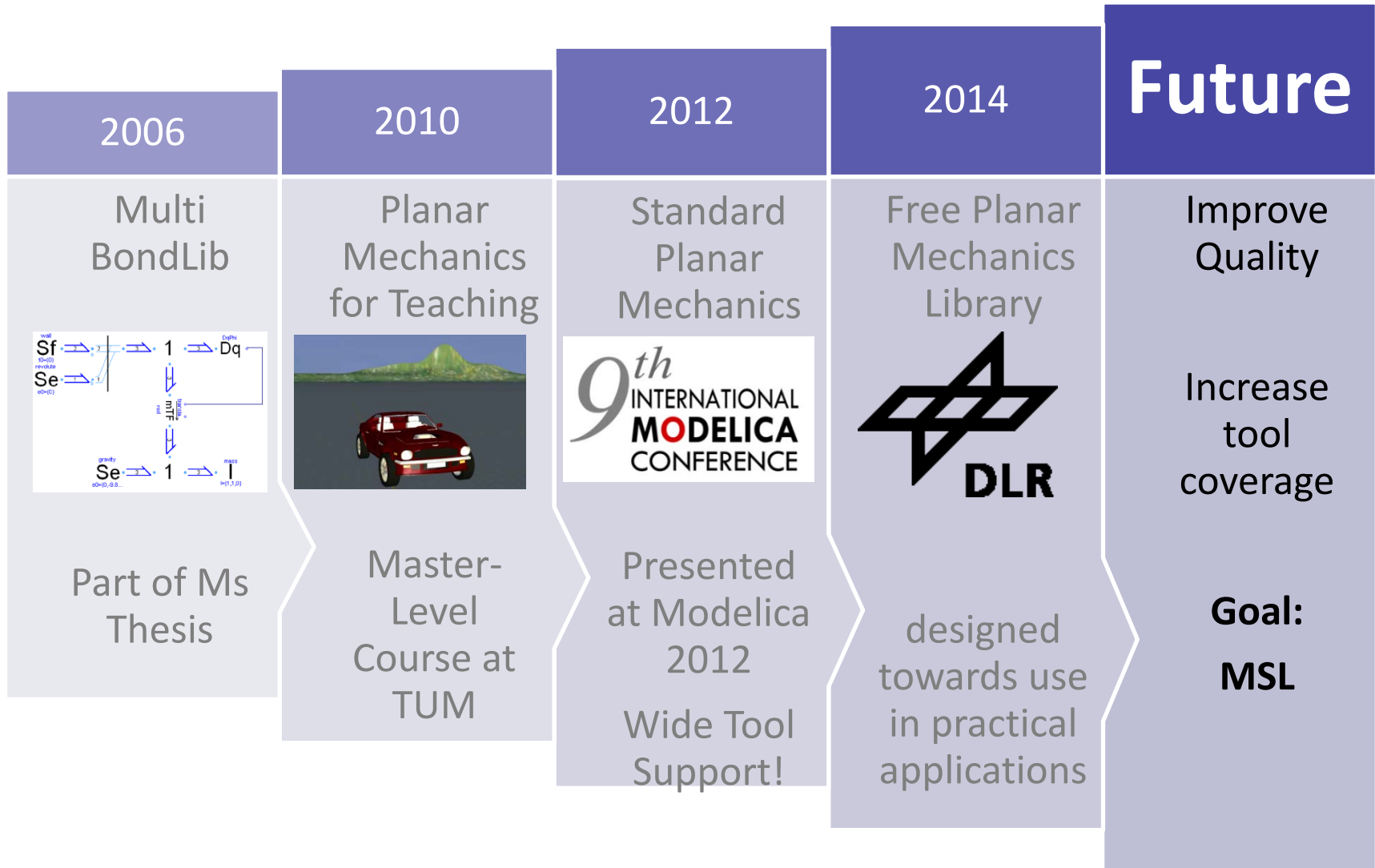
Components for Gears

- Ideal internal contact
- Ideal external contact

Examples:

- Spur Gear
- Planetary Gear





Questions ?