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Multiple Shooting with Collocation: An Efficient Approach to Dynamic Optimization, with Possibilities for Integration in OpenModelica

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Motivation: NMPC using multiple shooting with collocation

NMPC is superior to traditional control in many cases

Critical issues:

- Accuracy
- Problem size - computational effort
- Safety specifications – path constraints

 Fast algorithm is needed

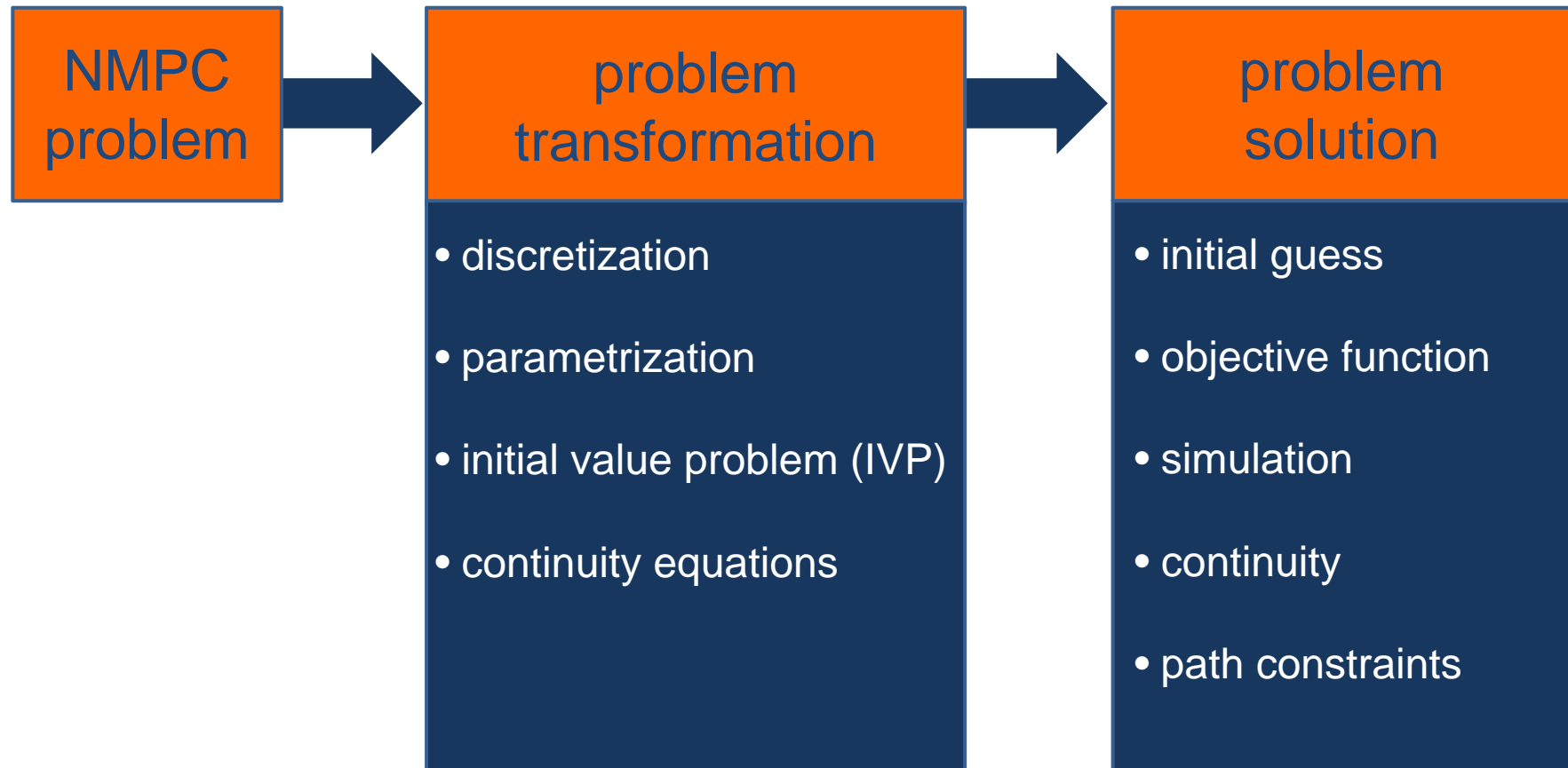
Nonlinear optimal control problem

$$\min_{u(t), x(t), y(t), p} \left\{ J = \Phi(x_f, y_f, t_f, p) + \int_{t_0}^{t_f} L(x(t), y(t), u(t), p) dt \right\}$$

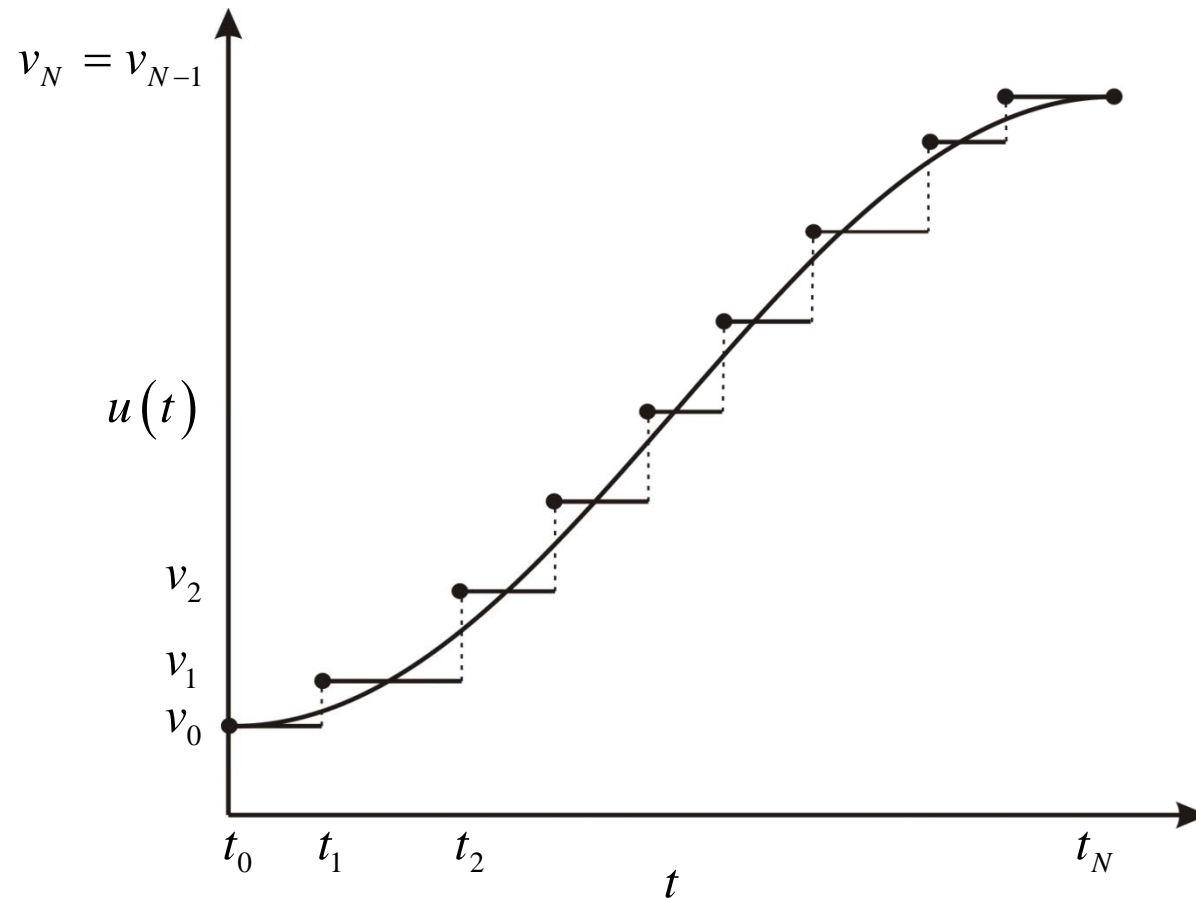
$$\begin{aligned} \text{s.t.} \quad & \dot{x}(t) = f(x(t), y(t), u(t), t, p) & x(t_0) = x_0 \\ & 0 = g(x(t), y(t), u(t), t, p) & t \in [t_0, t_f] \\ & 0 \leq h(x(t), y(t), u(t), t, p) \\ & 0 = r_e(x(t_i), y(t_i), p) \\ & 0 \leq r_i(x(t_i), y(t_i), p) \end{aligned}$$

Bounds of state variables, controls and parameters

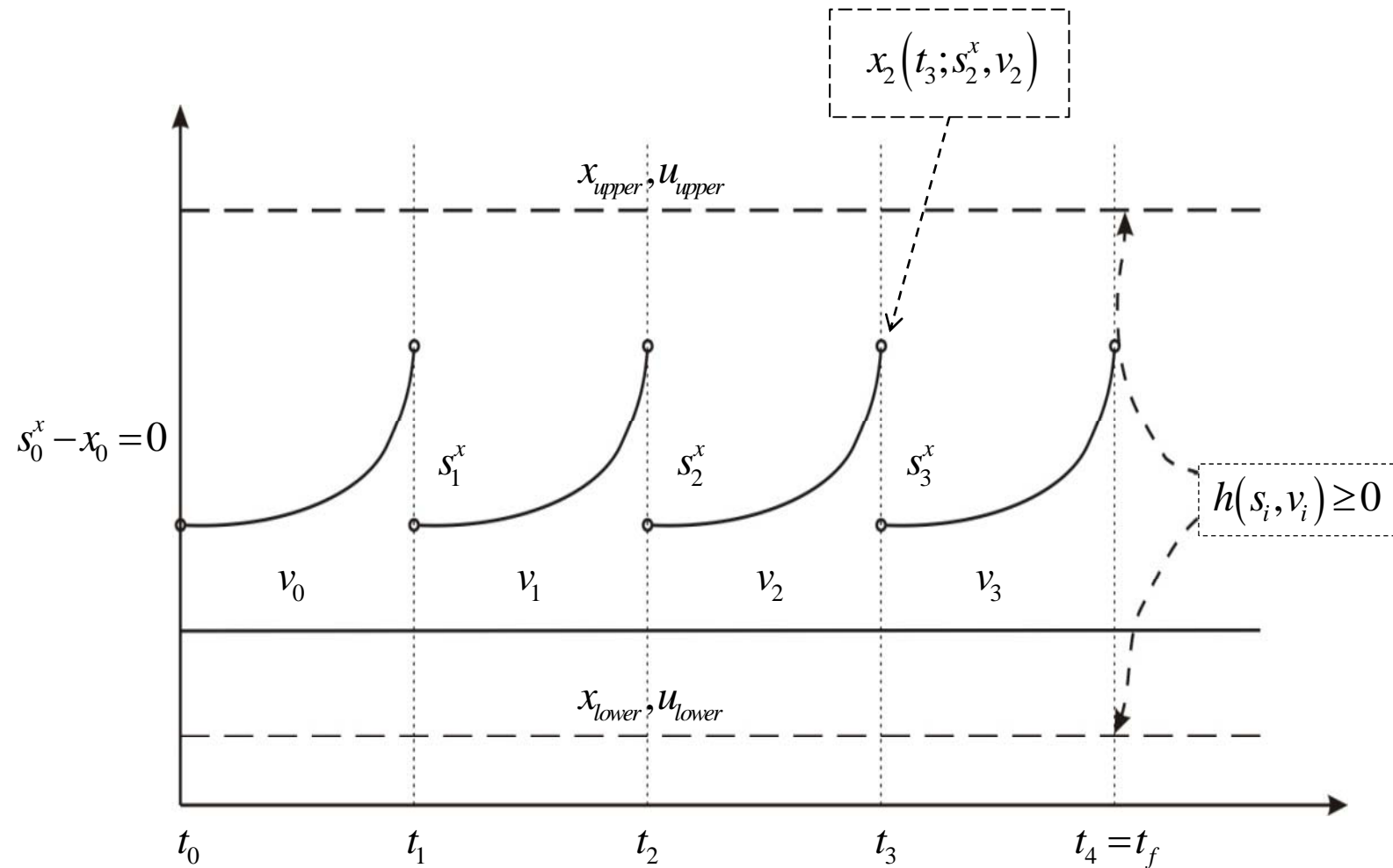
Multiple shooting



Multiple shooting – control parametrization



Multiple shooting – state parametrization

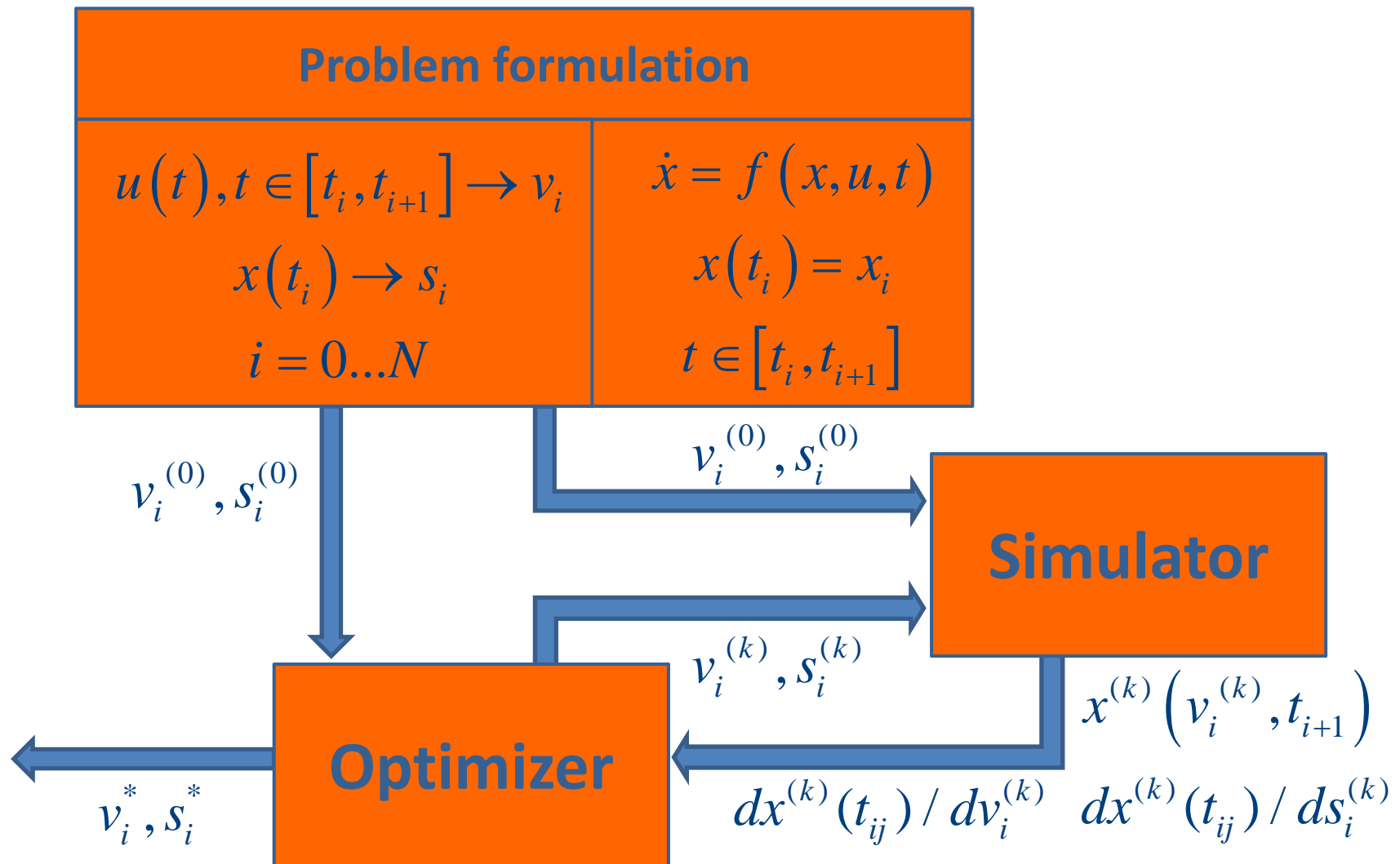


Finite dimensional NLP problem

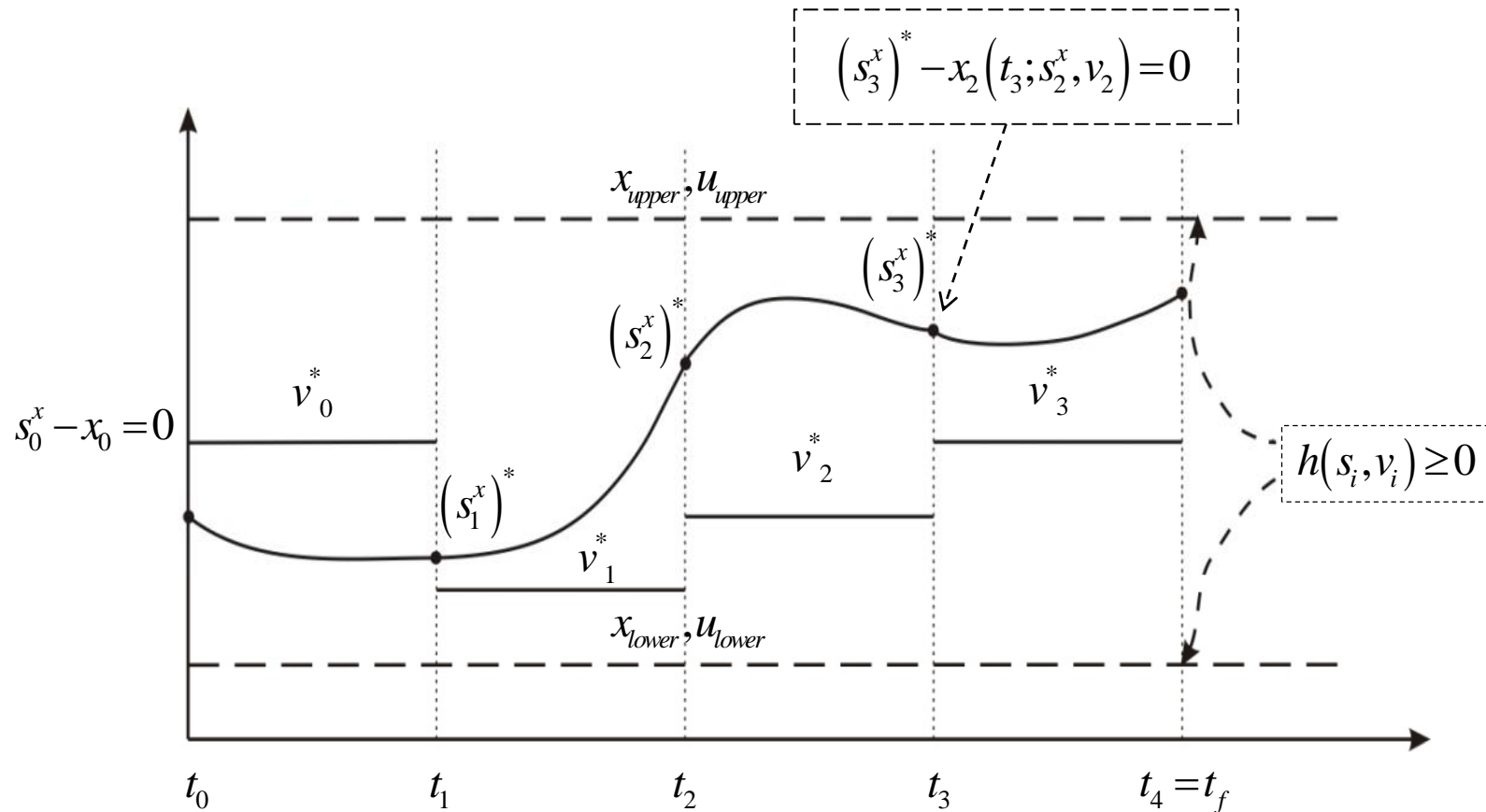
$$\min_{\substack{v_0, \dots, v_{N-1} \\ s_0, \dots, s_{N-1}}} \left\{ J = \Phi(s_N, p, t_N) + \sum_{i=0}^{N-1} L(s_i, v_i, p) \right\}$$

$$\begin{aligned} \text{s.t.} \quad & 0 = s_0 - x_0 \\ & 0 = s_{i+1} - x_i(s_i, v_i, p, t_{i+1}) && i = 0, 1, \dots, N-1 \\ & 0 = g(s_i, v_i, p) && i = 0, 1, \dots, N \\ & 0 \leq h(s_i, v_i, p) \\ & 0 = r_e(s_N, p) \\ & 0 \leq r_i(s_N, p) \end{aligned}$$

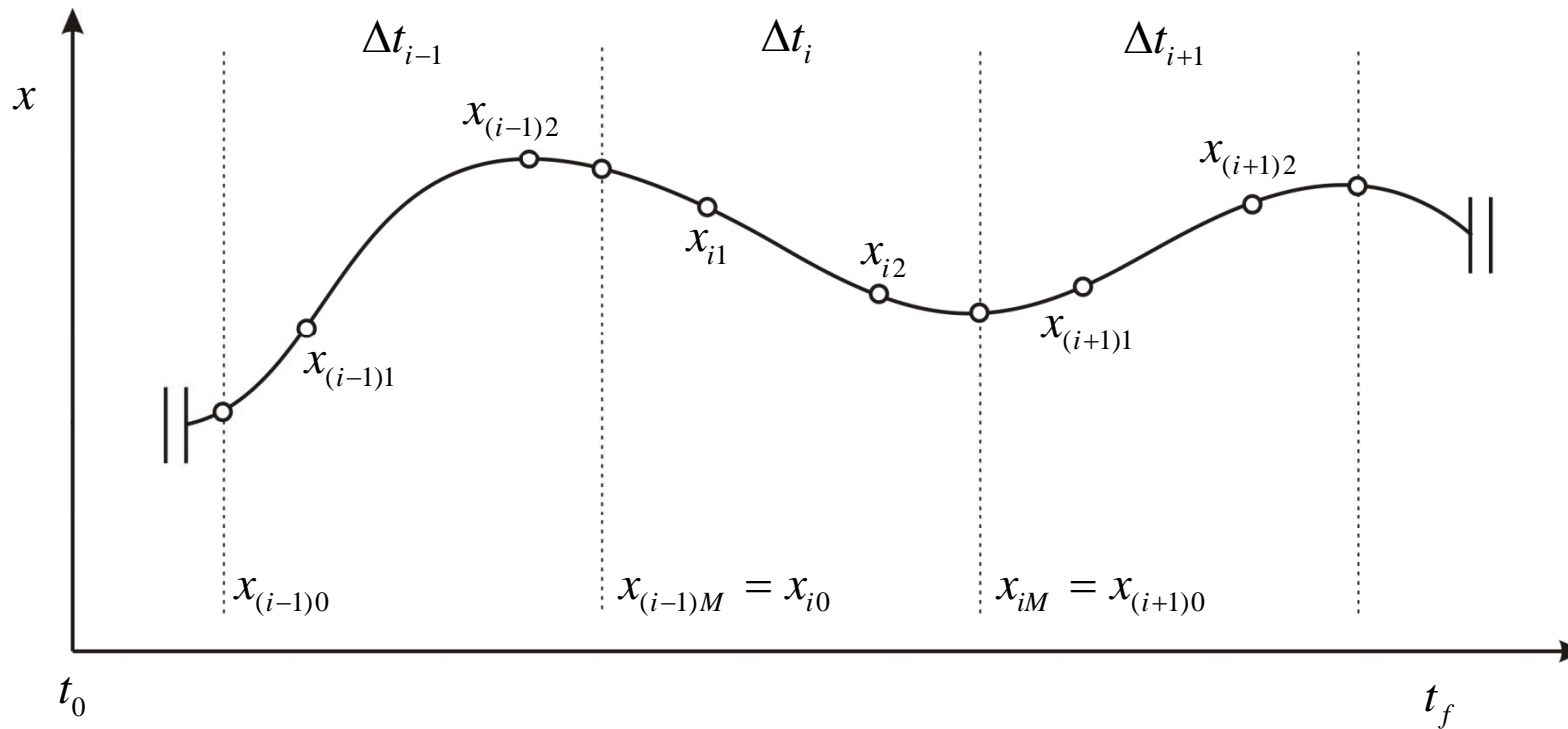
Multiple shooting



Multiple shooting - problem solution



Collocation on finite elements



Collocation on finite elements

$$x(t) = \sum_{k=0}^K x_{ik} l_k(\tau)$$

\downarrow

$$\frac{d}{dt} \quad \frac{d}{d\tau} \frac{d\tau}{dt}$$
$$\dot{x}(t) = \frac{1}{h_i} \sum_{k=0}^K x_{ik} \frac{dl_k(\tau)}{d\tau}$$

$$l_k(\tau) = \prod_{k \neq j=0}^K \frac{\tau - \tau_j}{\tau_k - \tau_j}$$

$$t \in [t_i, t_{i+1}]$$

$$t = t_i + h_i \tau$$

$$\dot{x}(t) = f(x, t)$$

$$\sum_{k=0}^K x_{ik} \frac{dl_k(\tau)}{d\tau} = h_i f(x, t)$$

Advantages of collocation on finite elements

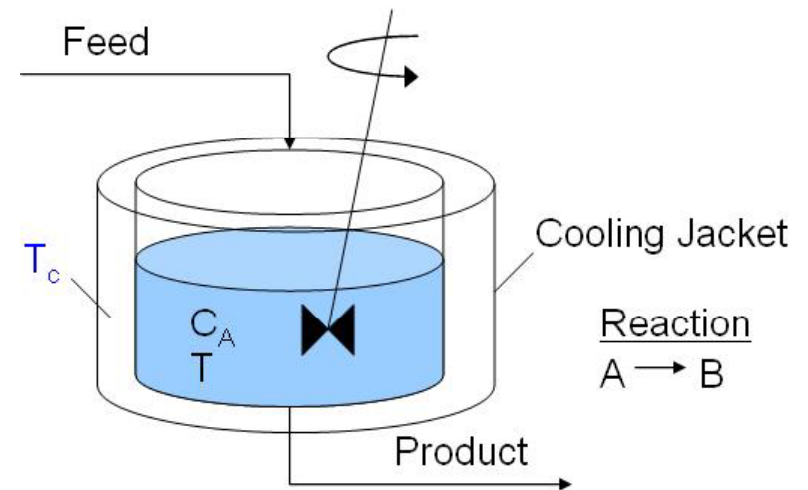
- **Single-step method** → smooth profiles required only *within* the finite elements, discontinuity of control profiles at $t_i, i = 1, \dots, N$ possible
- **High-order implicit method** → provides accurate profiles with relatively few finite elements, i.e. reduces problem size
- No stability limitations on element size for stiff systems
- Path constraints can be held inside the elements

Case study - CSTR

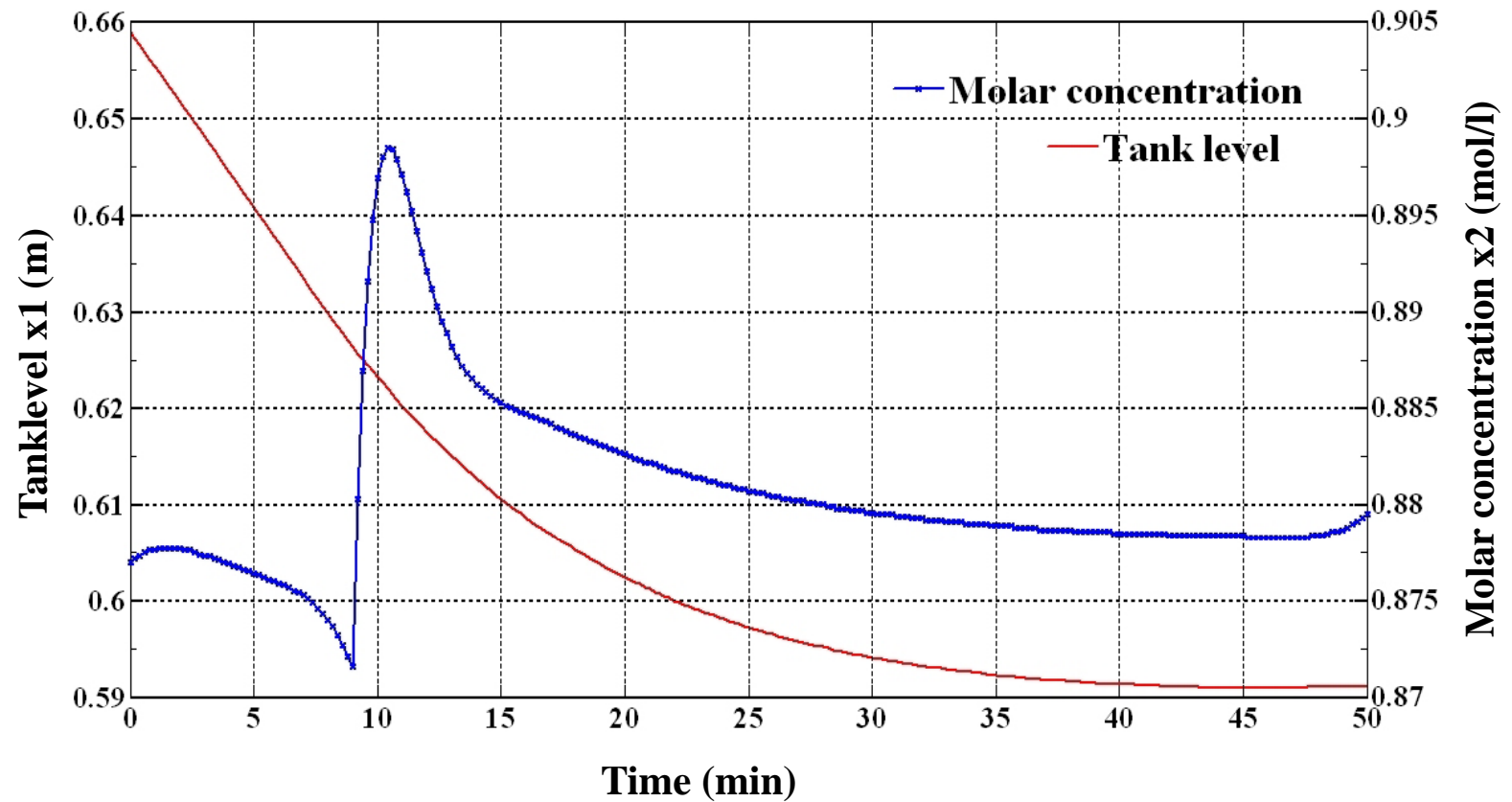
$$\min_{x,u} \int_{t_0}^{t_f} \sum_{i=1}^{N_x} w_i (x_i - x_i^{(ref)})^2 + \sum_{i=1}^{N_u} w_i (u_i - u_i^{(ref)})^2 dt$$

s.t. model equations
 variable bounds

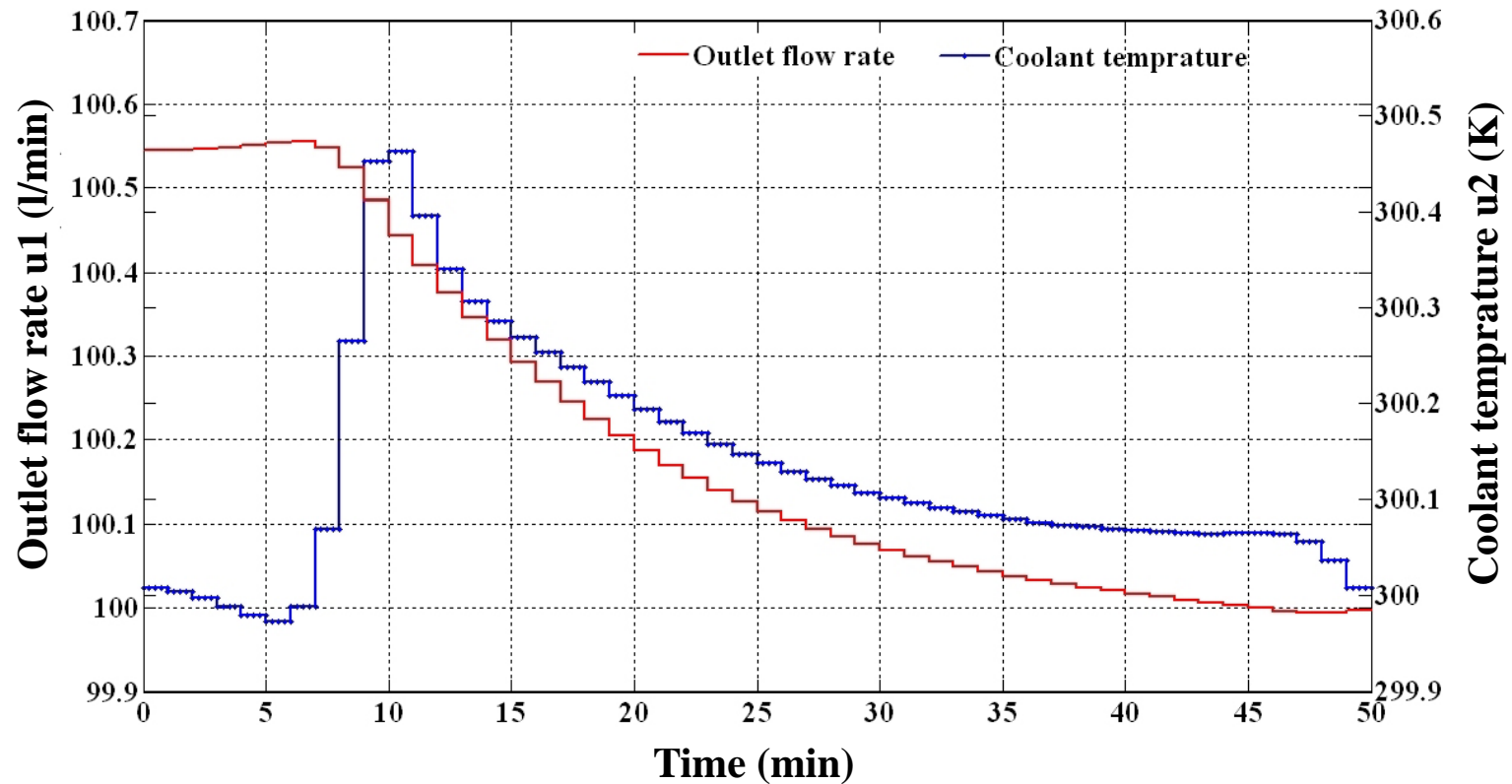
Using 50 subintervals,
IPOPT 3.4.0 optimizer.
CPU time is 0.953 sec.



Case study - CSTR



Case study - CSTR

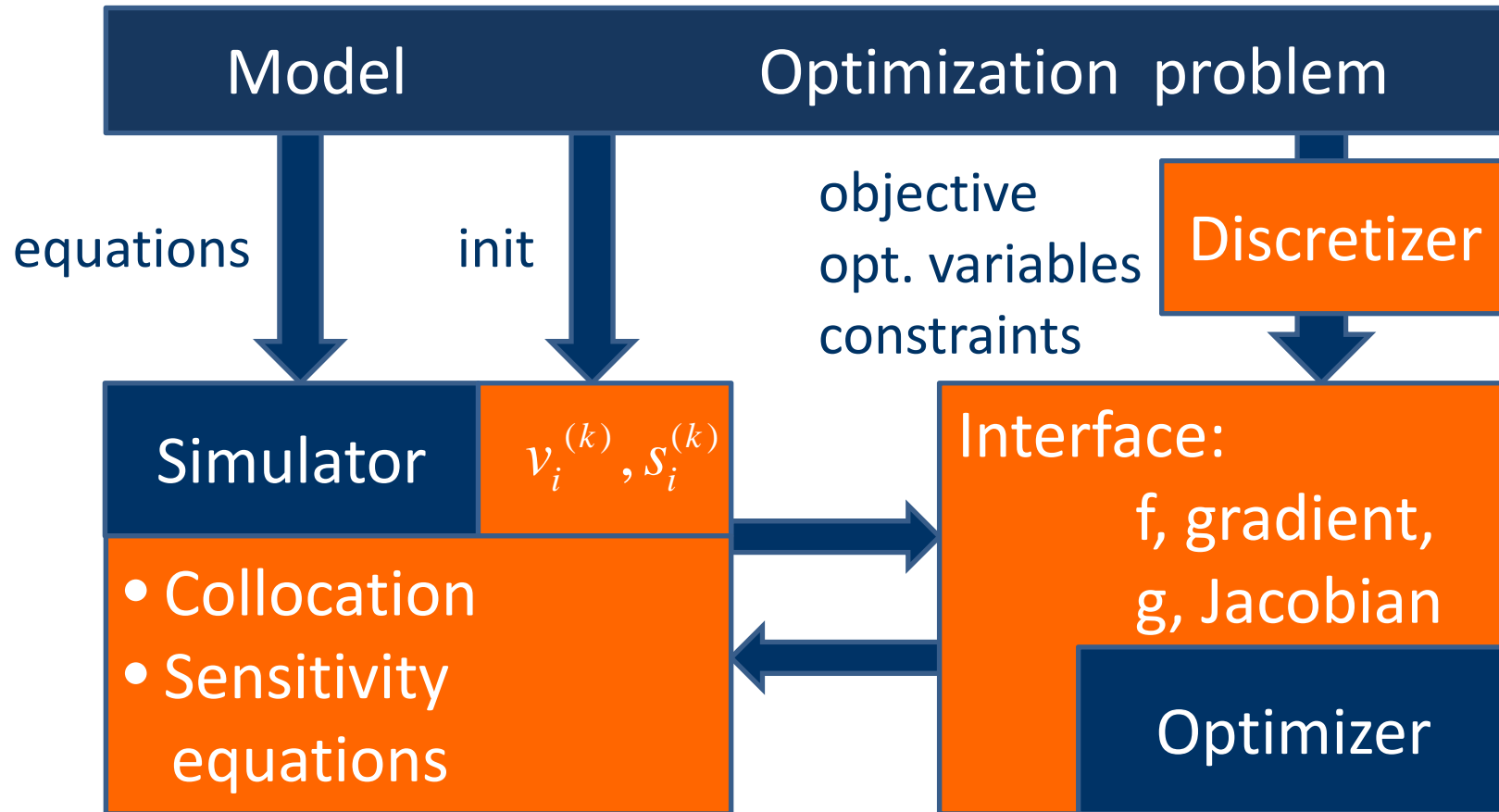


Classification of methods – consequences for the optimization framework

	Single shooting	Multiple shooting	Simultaneous approach
Optimization variables	v_i $i = 0, \dots, N - 1$	v_i, s_i $i = 0, \dots, N - 1$	v_i, x_i $i = 0, \dots, N \times K$
Problem size	small	intermediate	large
Simulation	1 IVP	N IVP`s	None
Path constraints	none	fulfilled at $t_i, i = 0, \dots, N \times K$	fulfilled at $t_i, i = 0, \dots, N \times K$

Implementation in OpenModelica

What is needed?



Summary

Collocation method: fast accurate simulator

Multiple shooting: reduced problem size, path constraints incorporated

Multiple shooting with collocation: high accuracy and reduced computational effort

Structure of the algorithm: reveals what is needed for implementation in OpenModelica



**Many thanks
for your attention!**

References

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- [3] L. T. Biegler, Solution of dynamic optimization problem by successive quadratic programming and orthogonal collocation, *Computers and Chemical Engineering* 8 (1984), 243-248.
- [4] D. B. Leineweber, H. G. Bock, J. P. Schlöder, An efficient multiple shooting based reduced SQP strategy for large-scale dynamic process optimization, *Computers and Chemical Engineering* 27(2), (2003), 157-166.